

2025 VCE Specialist Mathematics Examination 2 Solutions *with*

TI-Nspire™ CX II CAS
graphing calculator



Teachers Teaching with Technology™
Professional Development from Texas Instruments

$$\frac{d}{dx}(\sin^{-1}(ax)) = \frac{a}{\sqrt{1-a^2x^2}}$$

calculus

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

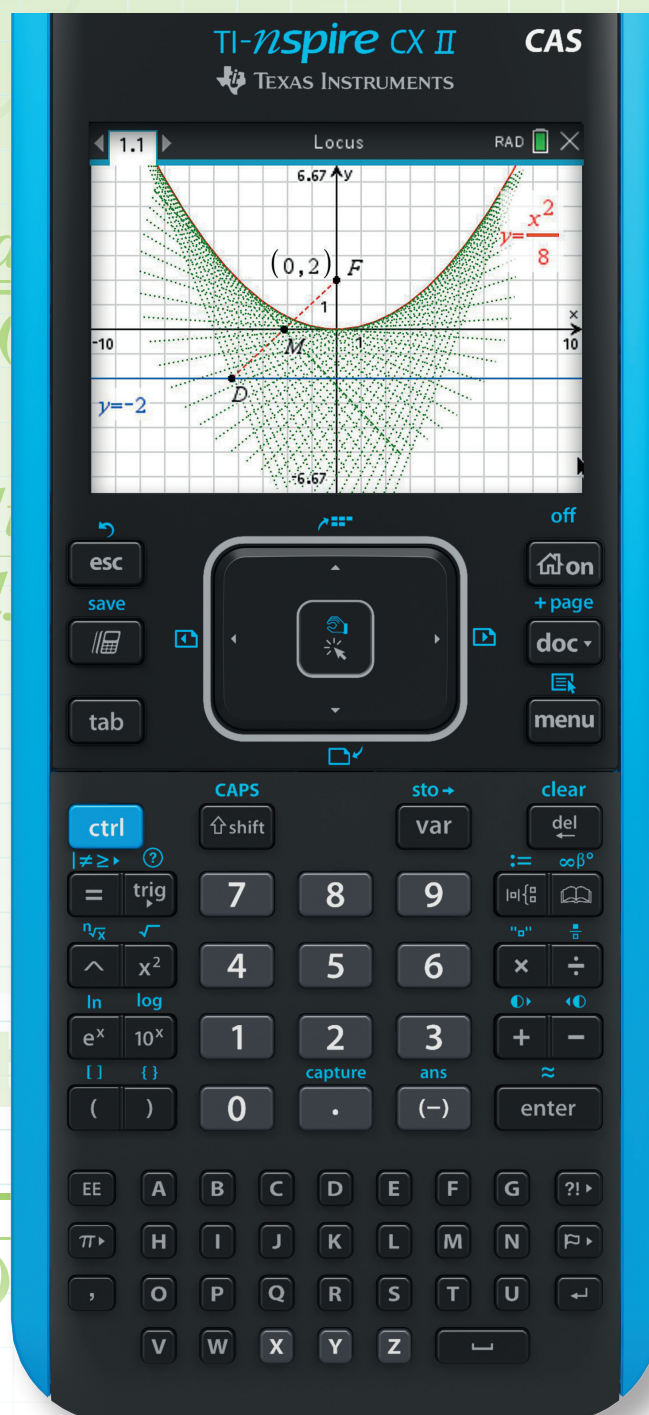
kinematics

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} =$$

circular functions

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

For more VCE Resources, visit:
www.education.ti.com/aus/vic



Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1

A tiger is a type of cat.

Consider the following statement.

‘If I have a tiger, then I have a cat.’

Switch order of statement and then negate

The **contrapositive** of this statement is

- A. if I do not have a tiger, then I do not have a cat.
- B. if I have a cat, then I have a tiger.
- C.** if I do not have a cat, then I do not have a tiger.
- D. if I do not have a tiger, then I have a different type of cat.

Question 2

Consider the following statement.

‘If $f''(0) = 0$, then the graph of f necessarily has a point of inflection at $x = 0$.’

A **counter-example** that disproves this statement is when

A. $f(x) = \sin^{-1}(x)$

B. $f(x) = \frac{2x}{x^2 - 1}$

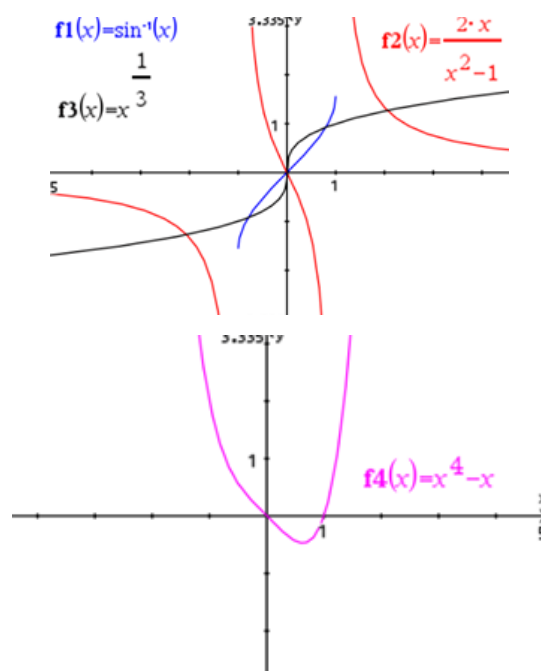
C. $f(x) = x^{\frac{1}{3}}$

D. $f(x) = x^4 - x$

A and B: $f''(0) = 0$ and concavity changes at $x = 0$

C: $f''(0)$ undefined

D: $f''(x) = 12x^2 \Rightarrow f''(0) = 0$
 $f''(x) \geq 0$ so concavity does not change.
 Hence, $x = 0$ is not an inflection point.



Question 3

The graph of $y = \frac{x^2 + a}{bx + c}$ has an asymptote given by $y = -\frac{1}{2}x + \frac{1}{4}$ and a y-intercept of -2 .

The values of a , b and c are

- A.** $a = 2, b = -2, c = -1$
B. $a = 2, b = 2, c = -1$
C. $a = -2, b = -2, c = 1$
D. $a = -2, b = -2, c = -1$

$$\text{propFrac}\left(\frac{x^2+a}{b \cdot x+c}, x\right) = \frac{a \cdot b^2+c^2}{b^2 \cdot (b \cdot x+c)} + \frac{x}{b} - \frac{c}{b^2}$$

$$\text{solve}\left(\frac{x^2+a}{b \cdot x+c} = -2, a\right) | x=0 \text{ and } b=-2 \text{ and } c=-1$$

$a=2$

Equation of non-vertical asymptote

$$y = \frac{x}{b} - \frac{c}{b^2}$$

$$\frac{1}{b} = -\frac{1}{2} \Rightarrow b = -2, -\frac{c}{4} = \frac{1}{4} \Rightarrow c = -1$$

$$\text{When } x=0, y = \frac{a}{c} = \frac{a}{-1} = -2 \Rightarrow a=2$$

Question 4

Consider the following algorithm used to estimate a volume of revolution.

```

define f(x)
  return  $\sqrt{x+1}$ 
sum  $\leftarrow$  0
a  $\leftarrow$  1
b  $\leftarrow$  3
left  $\leftarrow$  a
while left  $\leq$  b
  volume  $\leftarrow$   $\pi(f(\text{left}))^2$ 
  sum  $\leftarrow$  sum + volume
  left  $\leftarrow$  left + 1
end while
print sum

```

$$\text{sum} = 0 + \pi(f(1))^2 + \pi(f(2))^2 + \pi(f(3))^2 \text{ where } (f(x))^2 = x+1$$

$$\text{sum} = 0 + 2\pi + 3\pi + 4\pi = 9\pi$$

The algorithm above will print the value

- A.** 5π
B. 9π
C. 14π
D. 29π

$f(x) := \sqrt{x+1}$	Done
$0 + \pi \cdot ((f(1))^2 + (f(2))^2 + (f(3))^2)$	$9 \cdot \pi$

Question 5

The equation $z^3 + az^2 + bz - 52 = 0$, where $a, b \in \mathbb{R}$ and $z \in \mathbb{C}$, has a solution $z = 2 - 3i$.

The value of ab is

- A.** -232
B. -64
C. -8
D. 0

Conjugate root theorem applies, so $2 + 3i$ is also a root

$p(z) := z^3 + a \cdot z^2 + b \cdot z - 52$ Done
 solve $\left\{ \begin{array}{l} p(2-3i) = 0 \\ p(2+3i) = 0 \end{array} \right\}, \{a, b\}$
 $a = -8$, and $b = 29$.
 $-8 \cdot 29 = -232$

Question 6

Let $z \in \mathbb{C}$.

Given that $|z| = 1$ and $z \neq 1$, $\operatorname{Re}\left(\frac{1}{1-z}\right)$ is

- A.** $-\frac{1}{2}$
B. 0
C. $\frac{1}{2}$
D. $\frac{\sqrt{3}}{2}$

Polar form easier to navigate
 (calc must be in radian mode)

$\operatorname{real}\left(\frac{1}{1-e^{i \cdot \theta}}\right)$ $\frac{1}{2}$

Question 7

Using the substitution $u = \cos(\theta)$, $\frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin(2\theta)}{1+\cos(\theta)} d\theta$ can be expressed as

- A.** $\int_0^{\frac{\pi}{2}} u \sqrt{\frac{1-u}{1+u}} du$
B. $\int_0^1 \left(1 + \frac{1}{1+u}\right) du$
C. $\int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{1+u}\right) du$
D. $\int_0^1 \left(1 - \frac{1}{1+u}\right) du$

When $\theta = 0$, $u = 1$ and when $\theta = \frac{\pi}{2}$, $u = 0$.

So not **A** and not **C**.

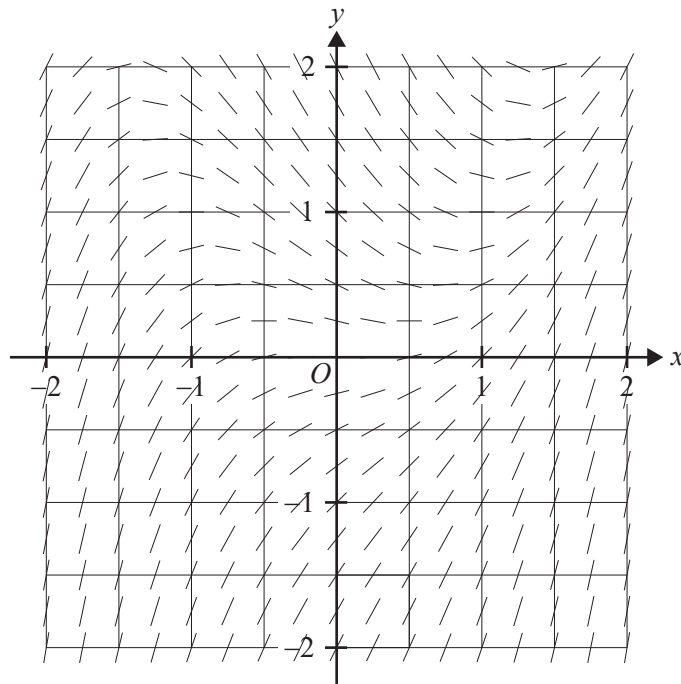
$$\int_0^{\frac{\pi}{2}} \frac{\sin(\theta) \cos(\theta)}{1+\cos(\theta)} d\theta$$

$$u = \cos(\theta) \Rightarrow \frac{du}{d\theta} = -\sin(\theta)$$

$$\int_1^0 \frac{-u}{1+u} du = \int_0^1 \frac{u}{1+u} du = \int_0^1 \frac{(1+u)-1}{1+u} du = \int_0^1 \left(1 - \frac{1}{1+u}\right) du$$

Question 8

Consider the direction field below.



The direction field best represents the differential equation

A. $\frac{dy}{dx} = x^2 - y$

B. $\frac{dy}{dx} = x - y^2$

C. $\frac{dy}{dx} = y - x$

D. $\frac{dy}{dx} = x - y$

At $(1, 0)$, $\frac{dy}{dx} \approx 1$ so not **C** $\left(\frac{dy}{dx} = -1\right)$.

At $(0, -1)$, $\frac{dy}{dx} \approx 1$ so not **B** $\left(\frac{dy}{dx} = -1\right)$.

At $(-1, 0)$, $\frac{dy}{dx} \approx 1$ so not **D** $\left(\frac{dy}{dx} = -1\right)$.

Question 9

A parametric curve is given by $x = kt$, $y = e^{kt}$, where k is a positive constant. The curve is rotated about the x -axis from $t = a$ to $t = b$, where $b > a$, to form a surface of revolution.

The area of this surface is given by

A. $2\pi \int_a^b e^{kt} \sqrt{k^2 t^2 + e^{2kt}} dt$

B. $2\pi \sqrt{k} \int_a^b e^{kt} \sqrt{1 + e^{kt}} dt$

C. $2\pi \int_{e^{ka}}^{e^{kb}} \sqrt{1 + u^2} du$

D. $2\pi \int_{e^{ka}}^{e^{kb}} u \sqrt{1 + u^2} du$

$$\frac{dx}{dt} = k \text{ and } \frac{dy}{dt} = ke^{kt}$$

$$S = 2\pi \int_a^b e^{kt} \sqrt{k^2 + k^2 e^{2kt}} dt$$

$$\text{Let } u = e^{kt} \text{ so } u^2 = e^{2kt} \text{ and } \frac{1}{k} \frac{du}{dt} = e^{kt}.$$

$$\text{When } t = a, u = e^{ka} \text{ and when } t = b, u = e^{kb}.$$

$$\begin{aligned} S &= 2\pi k \left(\frac{1}{k} \right) \int_{e^{ka}}^{e^{kb}} \sqrt{1 + u^2} du \\ &= 2\pi \int_{e^{ka}}^{e^{kb}} \sqrt{1 + u^2} du \end{aligned}$$

Question 10

The region bounded by the curve given by $y = 3\cos^{-1}(x)$, for $0 \leq y \leq a$, where $a > 0$, and the line $x = 0$ is rotated about the y -axis to form a solid of revolution. The volume of the solid is $\frac{\pi(4\pi + 3\sqrt{3})}{8}$.

The value of a is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. π

$$y = 3\cos^{-1}(x) \Rightarrow x = \cos\left(\frac{y}{3}\right)$$

$$V = \pi \int_0^a \cos^2\left(\frac{y}{3}\right) dy = \frac{\pi \left(3 \sin\left(\frac{2a}{3}\right) + 2a \right)}{4}$$

$$\frac{3 \sin\left(\frac{2a}{3}\right) + 2a}{4} = \frac{4\pi + 3\sqrt{3}}{8}$$

$$a = \pi \text{ (via solve)}$$

$$\pi \cdot \int_0^a \left(\cos\left(\frac{y}{3}\right) \right)^2 dy = \frac{\left(3 \cdot \sin\left(\frac{2 \cdot a}{3}\right) + 2 \cdot a \right) \cdot \pi}{4}$$

$$\Delta \text{ solve } \left(\frac{\left(3 \cdot \sin\left(\frac{2 \cdot a}{3}\right) + 2 \cdot a \right) \cdot \pi}{4} \right) = \frac{\pi \cdot (4 \cdot \pi + 3 \cdot \sqrt{3})}{8}$$

$$a = 3.14159$$

Question 11

Menu > Calculus > Differential Equation Solver

Given that $y(x)$ is a solution to the differential equation $\frac{dy}{dx} = x^2 y^3$, where $y(1) = 3$, the domain of y is

A. $x \leq \left(\frac{7}{6}\right)^{\frac{1}{3}}$

B. $x < \left(\frac{7}{6}\right)^{\frac{1}{3}}$

C. $x \geq \left(\frac{7}{6}\right)^{\frac{1}{3}}$

D. $x > \left(\frac{7}{6}\right)^{\frac{1}{3}}$

Solving the DE gives $\frac{1}{18} - \frac{1}{2y^2} = \frac{x^3}{3} - \frac{1}{3}$.

Solving this equation gives $y = \pm 3 \sqrt{\frac{-1}{6x^3 - 7}}$.

Solving $\frac{-1}{6x^3 - 7} > 0$ for x gives $x < \left(\frac{7}{6}\right)^{\frac{1}{3}}$.

$$\text{deSolve}(y' = x^2 \cdot y^3 \text{ and } y(1) = 3, x, y)$$

$$\frac{1}{18} - \frac{1}{2 \cdot y^2} = \frac{x^3}{3} - \frac{1}{3}$$

Differential Equation Solver

Equation: $y = x^2 y^3$

Example: $y' = 2y$

Independent Var: x

Dependent Var: y

Condition: $y(1) = 3$

Condition: (Optional)

Example: $y(0) = 1$

OK Cancel

$$\text{solve}\left(\frac{1}{18} - \frac{1}{2 \cdot y^2} = \frac{x^3}{3} - \frac{1}{3}, y\right)$$

$$y = 3 \cdot \sqrt{\frac{-1}{6 \cdot x^3 - 7}} \text{ and } \frac{1}{6 \cdot x^3 - 7} \leq 0 \text{ or } y = -3 \cdot \sqrt{\frac{-1}{6 \cdot x^3 - 7}}$$

$$\text{solve}\left(\frac{-1}{6 \cdot x^3 - 7} > 0, x\right)$$

$$x < \frac{7^{\frac{1}{3}} \cdot 6^{\frac{2}{3}}}{6}$$

Question 12

A particle moves along a straight line with **constant acceleration**. It passes through a point A with velocity $u \text{ m s}^{-1}$ and then through a point B with velocity $v \text{ m s}^{-1}$.

The velocity of the particle at the **midpoint** of the line segment AB is given by

A. $\frac{u+v}{2}$

Let v_m be the velocity of the particle at the midpoint.

B. $u + \frac{u+v}{2}$

Let $\frac{s}{2}$ be the distance travelled to reach the midpoint.

C. $\frac{u^2 + v^2}{2}$

$$v^2 = u^2 + 2as \Rightarrow a = \frac{v^2 - u^2}{2s}$$

$$v_m^2 = u^2 + 2 \left(\frac{v^2 - u^2}{2s} \right) \left(\frac{s}{2} \right)$$

$$= u^2 + \frac{v^2 - u^2}{2}$$

$$= \frac{u^2 + v^2}{2}$$

D. $\sqrt{\frac{u^2 + v^2}{2}}$

$$\text{so } v_m = \sqrt{\frac{u^2 + v^2}{2}}, \text{ assuming } v_m > 0$$

$$\text{solve}(v^2 = u^2 + 2 \cdot a \cdot s, a)$$

$$a = \frac{-(u^2 - v^2)}{2 \cdot s}$$

$$\text{solve}\left(v_m^2 = u^2 + 2 \cdot \frac{-(u^2 - v^2)}{2 \cdot s} \cdot \frac{s}{2}, v_m\right) | v_m > 0$$

$$v_m = \frac{\sqrt{2 \cdot (u^2 + v^2)}}{2} \text{ and } \sqrt{u^2 + v^2} > 0$$

Question 13

From an open window, a person projects a ball **vertically up** using an outstretched arm so the ball does not strike any part of the building. The point of projection of the ball is **50 m above the ground** and its **velocity of projection is 20 m s^{-1}** .

The **time**, in seconds, it takes for the ball to reach the tray of a truck that is **1 m above the ground** directly below the point of projection is closest to

A. 1.72

Take upwards direction as positive.

B. 5.80

$$u = 20, s = -(50 - 1), a = -9.8, t = ?$$

C. 5.83

$$\text{Using } s = ut + \frac{1}{2}at^2$$

D. 1.75

$$\text{Solving } -(50 - 1) = 20t - \frac{9.8}{2}t^2 \text{ for } t \text{ with } t > 0 \text{ gives}$$

$$t = 5.80444...$$

$$\text{solve}\left(-(50 - 1) = 20 \cdot t - \frac{9.8}{2} \cdot t^2, t\right) | t > 0$$

$$t = 5.80445$$

Question 14

For non-zero vectors $\underline{a}$ and $\underline{b}$, if $\underline{a} \cdot \underline{b} = |\underline{a} \times \underline{b}|$, then the angle between $\underline{a}$ and $\underline{b}$ is

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

$$ab \cos(\theta) = ab \sin(\theta)$$

$$\tan(\theta) = 1 \quad (ab \neq 0)$$

$$\theta = \frac{\pi}{4}$$

Question 15

Consider the two planes described by the equations $2x + 2y + z = 2$ and $ax + 4z = 1$, where a is a positive constant.

The angle between the two planes is $\cos^{-1}\left(\frac{2}{3}\right)$.

The value of a satisfies the equation

A. $a + 2 = \sqrt{a^2 + 16}$

Menu > Matrix & Vector > Norms > Norm to find the magnitude of a vector.
Menu > Matrix & Vector > Vector > Dot Product to find the dot product.

B. $\frac{2a+4}{3\sqrt{a^2+16}} = \frac{3}{2}$

C. $2a+4 = 3\sqrt{a^2+16}$

D. $\frac{2a+4}{\sqrt{a^2+16}} = \frac{2}{3}$

$$\cos(\theta) = \frac{\underline{n}_1 \cdot \underline{n}_2}{|\underline{n}_1||\underline{n}_2|}$$

$$\frac{2}{3} = \frac{(2\underline{i} + 2\underline{j} + \underline{k}) \cdot (a\underline{i} + 4\underline{k})}{\sqrt{2^2 + 2^2 + 1^2} \times \sqrt{a^2 + 4^2}}$$

$$\frac{2}{3} = \frac{2(a+2)}{3\sqrt{a^2+16}}$$

$$a+2 = \sqrt{a^2+16}$$

$$\frac{\text{dotP}([2 \ 2 \ 1], [a \ 0 \ 4])}{\text{norm}([2 \ 2 \ 1]) \cdot \text{norm}([a \ 0 \ 4])} = \frac{2 \cdot (a+2)}{3 \cdot \sqrt{a^2+16}}$$

Question 16

The position vector of a particle at time t is given by $\underline{r}(t) = ne^{-2t}\underline{i} - t^2\underline{j}$, where n is a positive constant.

For what value of n is the particle's acceleration perpendicular to its velocity when $t = \frac{1}{2}$?

A. $2e$

B. $\frac{e^{0.5}}{2}$

C. $\frac{e}{2}$

D. $\frac{e}{2\sqrt{2}}$

Menu > Matrix & Vector > Vector > Dot Product
shift and then the addition key to access the Derivative template

$$\underline{r}(t) := [n \cdot e^{-2 \cdot t} \quad -t^2]$$

$$\text{dotP}\left(\frac{d}{dt}(\underline{r}(t)), \frac{d^2}{dt^2}(\underline{r}(t))\right) = 4 \cdot t - 8 \cdot n^2 \cdot e^{-4 \cdot t}$$

$$\text{solve}(4 \cdot t - 8 \cdot n^2 \cdot e^{-4 \cdot t} = 0, n) \Big|_{t=\frac{1}{2}} \text{ and } n > 0$$

$$n = \frac{e}{2}$$

Question 17

The **acceleration vector** of a particle that **starts from rest** is given by

$$\underline{a}(t) = 4\cos(2t)\underline{i} + 10\sin(2t)\underline{j} - 6e^{-2t}\underline{k}, \text{ where } t \geq 0.$$

The **velocity vector** of the particle, $\underline{v}(t)$, is given by

- A. $\underline{v}(t) = 2\sin(2t)\underline{i} - 5\cos(2t)\underline{j} + 3e^{-2t}\underline{k}$
- B. $\underline{v}(t) = 2\sin(2t)\underline{i} - 5(\cos(2t) + 1)\underline{j} + 3(e^{-2t} + 1)\underline{k}$
- C.** $\underline{v}(t) = 2\sin(2t)\underline{i} - 5(\cos(2t) - 1)\underline{j} + 3(e^{-2t} - 1)\underline{k}$
- D. $\underline{v}(t) = -8\sin(2t)\underline{i} + 20\cos(2t)\underline{j} + 12e^{-2t}\underline{k}$

Press shift and then the subtraction key to access the Integral template.

Handwritten solution for Question 17:

$$\underline{v}(t) := \int \underline{a}(t) dt + \underline{c}$$

Done

$$\underline{a} := \begin{bmatrix} 4 \cdot \cos(2 \cdot t) & 10 \cdot \sin(2 \cdot t) & -6 \cdot e^{-2 \cdot t} \end{bmatrix}$$

$$\underline{v} := \int \underline{a} dt$$

$$\underline{c} := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} - \underline{v}|_{t=0} \quad \begin{bmatrix} 0 & 5 & -3 \end{bmatrix}$$

$$\underline{v} + \underline{c}$$

$$\begin{bmatrix} 2 \cdot \sin(2 \cdot t) & 5 - 5 \cdot \cos(2 \cdot t) & 3 \cdot e^{-2 \cdot t} - 3 \end{bmatrix}$$

Question 18

The lines given by $\underline{r}_1(\lambda) = 2\underline{i} + r\underline{j} - 3\underline{k} + \lambda(\underline{i} - \underline{j} + 4\underline{k})$ and $\underline{r}_2(\mu) = \underline{i} + s\underline{k} + \mu(\underline{i} + \underline{j} - \underline{k})$ intersect at the point $(4, 3, t)$, where $\lambda, \mu \in \mathbb{R}$ and r, s and t are real constants.

The values of r, s and t respectively are

- A. 2, 3 and 5
- B. 5, 3 and 5
- C. 5, 5 and 8
- D.** 5, 8 and 5

From $\underline{r}_1(\lambda)$:

$$2 + \lambda = 4 \Rightarrow \lambda = 2; r - 2 = 3 \Rightarrow r = 5; t = -3 + 8 = 5$$

From $\underline{r}_2(\mu)$:

$$1 + \mu = 4 \Rightarrow \mu = 3; s - 3 = 5 \Rightarrow s = 8$$

Question 19

The plane with equation $x + y + z = a$, where $a \in \mathbb{R}$, intersects the coordinate axes at three points that form the vertices of a triangle.

The area of this triangle is given by

- A. $\frac{a^2\sqrt{3}}{4}$
- B.** $\frac{a^2\sqrt{3}}{2}$
- C. $\frac{a^2}{4}$
- D. $a^2\sqrt{3}$

Equilateral triangle of side length $a\sqrt{2}$.

(A sketch of the triangle may assist).

Handwritten formula for the area of an equilateral triangle:

$$\frac{1}{2} \cdot a \cdot \sqrt{2} \cdot a \cdot \sqrt{2} \cdot \sin\left(\frac{\pi}{3}\right) = \frac{a^2 \cdot \sqrt{3}}{2}$$

Question 20

Let $P \sim N(-2, 2^2)$, $Q \sim N(3, 3^2)$, $R \sim N(5, 6^2)$ and $Z \sim N(0, 1)$.

Given that P , Q and R are independent random variables, $\Pr(3P + 2Q - R > 25)$ is equal to

A. $\Pr\left(Z > \frac{5\sqrt{3}}{3}\right)$

B. $\Pr(Z > 5)$

C. $\Pr\left(Z > \frac{5\sqrt{66}}{11}\right)$

D. $\Pr\left(Z > \frac{30}{7}\right)$

$$\begin{aligned}
 X &= 3P + 2Q - R \\
 E(X) &= 3E(P) + 2E(Q) - E(R) \\
 &= 3(-2) + 2(3) - 5 \\
 &= -5 \\
 \text{var}(X) &= 3^2 \text{var}(P) + 2^2 \text{var}(Q) + \text{var}(R) \\
 &= 9(2)^2 + 4(3)^2 + 6^2 \\
 &= 108 \\
 X &\sim N\left(-5, (\sqrt{108})^2\right) \\
 \Pr(X > 25) &= \Pr\left(Z > \frac{25 - (-5)}{\sqrt{108}}\right) \\
 &= \Pr\left(Z > \frac{5\sqrt{3}}{3}\right)
 \end{aligned}$$

$$\frac{25 - (-5)}{\sqrt{108}}$$

$$\frac{5 \cdot \sqrt{3}}{3}$$

Do not write in this area.

Section B

Instructions

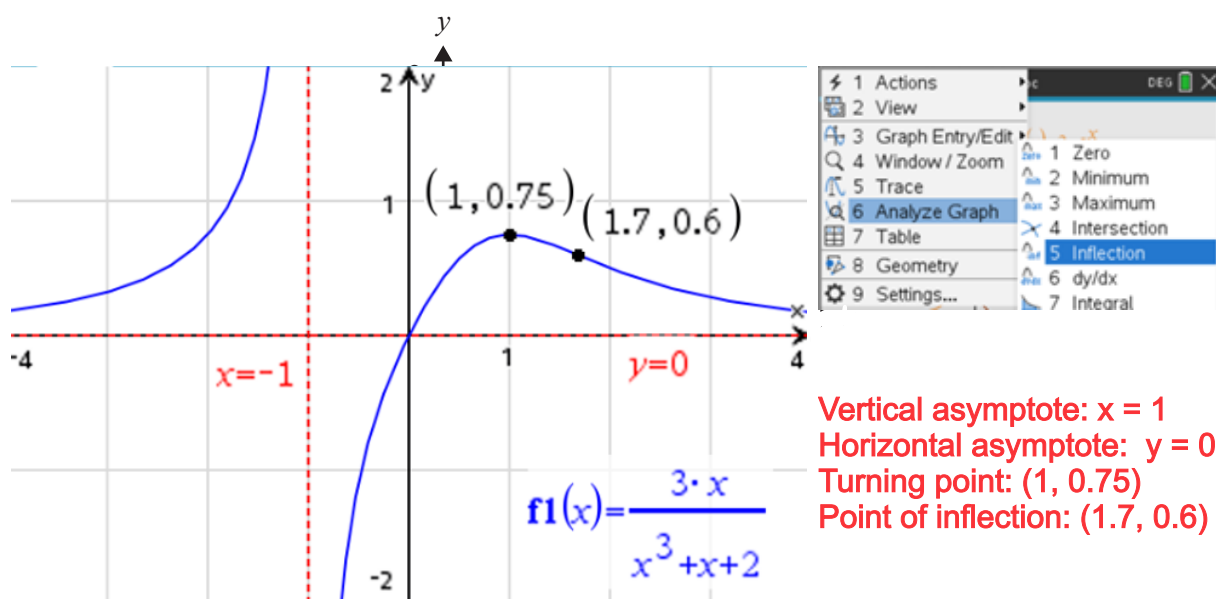
- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required for each question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1 (10 marks)

- a. Sketch the graph of $y(x) = \frac{3x}{x^3 + x + 2}$ on the axes below.

Label the **asymptotes with their equations**, and label the **turning point** and the **point of inflection** with their **coordinates**. Give the coordinates of the point of inflection correct to **one decimal place**.

3 marks



- b. The region bounded by the graph of $y = \frac{3x}{x^3 + x + 2}$, the coordinate axes and the line $x = 2$ is rotated about the **x-axis** to form a **solid of revolution**.

- i. **Write down a definite integral** that, when evaluated, will give the volume of the solid of revolution.

1 mark

$$V = \pi \int_0^2 \left(\frac{3x}{x^3 + x + 2} \right)^2 dx$$

- ii. Find the volume of the solid of revolution correct to two decimal places.

1 mark

$$V = 2.29 \text{ (correct to 2 decimal places).}$$

$$\pi \cdot \int_0^2 (f(x))^2 dx = 2.28801$$

$$\text{round}(2.2880103345827, 2) = 2.29$$

- c. Find the equations of the vertical asymptotes of the curve given by $y = \frac{3x}{x^3 - 5x + 2}$.

1 mark

The vertical asymptotes occur when the denominator equals zero.

Solving $x^3 - 5x + 2 = 0$ for x gives

$$x = 2, x = \sqrt{2} - 1, x = -\sqrt{2} - 1$$

$$\text{solve}(x^3 - 5 \cdot x + 2 = 0, x)$$

$$x = -(\sqrt{2} + 1) \text{ or } x = \sqrt{2} - 1 \text{ or } x = 2$$

- d. A family of curves is given by $y(x) = \frac{3x}{x^3 + ax + 2}$, where $a \in \mathbb{R}$.

- i. Consider the case where the graph has a stationary point P .

Find the y -coordinate of P in terms of a .

Press shift and then the subtraction key to access the Integral template.

1 mark

$$y'(x) = \frac{-6(x^3 - 1)}{(x^3 + ax + 2)^2}$$

$$y'(x) = 0 \text{ at } x = 1$$

$$y(1) = \frac{3}{a+3}$$

$$y(x) := \frac{3 \cdot x}{x^3 + a \cdot x + 2}$$

$$\Delta \text{ solve}\left(\frac{d}{dx}(y(x)) = 0, x\right) \quad x = 1$$

$$y(x)|_{x=1} \quad \frac{3}{a+3}$$

- ii. For a given value of a , the graph has no stationary points.

Find the equations of the vertical asymptotes of the graph in this case.

1 mark

If $a = -3$, there must be no stationary points.

Solving $x^3 - 3x + 2 = 0$ for x gives vertical asymptotes at $x = -2$ and $x = 1$.

$$\text{solve}(x^3 - 3 \cdot x + 2 = 0, x) \quad x = -2 \text{ or } x = 1$$

- iii. For a given value of a , the graph will have a point of inflection at $x = 2$.

Find the value of a .

2 marks

$$y''(x) = \frac{6(3x^5 - ax^3 - 12x^2 - 2a)}{(x^3 + ax + 2)^3}$$

$$y''(2) = \frac{-3(5a - 24)}{2(a + 5)^3}$$

$$\text{Solving } y''(2) = 0 \text{ for } a \text{ gives } a = \frac{24}{5}$$

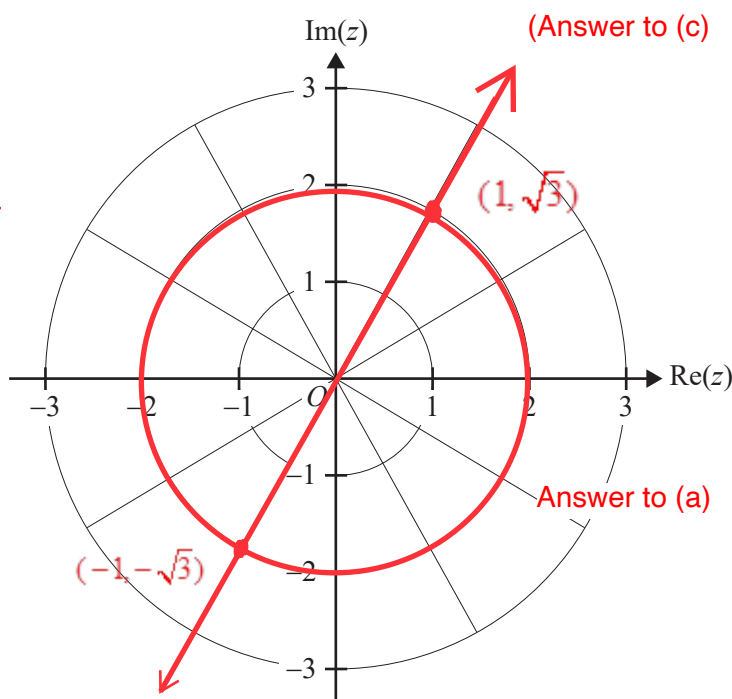
$$\Delta \frac{d^2}{dx^2}(y(x))|_{x=2} \quad \frac{-3 \cdot (5 \cdot a - 24)}{2 \cdot (a + 5)^3}$$

$$\text{solve}\left(\frac{-3 \cdot (5 \cdot a - 24)}{2 \cdot (a + 5)^3} = 0, a\right) \quad a = \frac{24}{5}$$

Question 2 (10 marks)

- a. Sketch $\{z : z\bar{z} = 4, z \in \mathbb{C}\}$ on the Argand plane below.

1 mark



- b. i. Show that $\{z : |z - 2i| = |z - \sqrt{3} - i|, z \in \mathbb{C}\}$ may be expressed as $y = \sqrt{3}x$.

2 marks

An algebraic approach can be used.

$$x^2 + (y - 2)^2 = (x - \sqrt{3})^2 + (y - 1)^2$$

$$x^2 + y^2 - 4y + 4 = x^2 - 2\sqrt{3}x + 3 + y^2 - 2y + 1$$

$$-2y = -2\sqrt{3}x$$

$$y = \sqrt{3}x$$

- ii. Sketch $\{z : |z - 2i| = |z - \sqrt{3} - i|, z \in \mathbb{C}\}$ on the Argand plane in **part a**.

1 mark

- c. i. Find the points of intersection of the curves defined in **part a** and in **part b.i**, expressing your answers in the form $a + ib$, where $a, b \in \mathbb{R}$.

2 marks

Solving $x^2 + y^2 = 4$ and $y = \sqrt{3}x$ for x and y

gives $x = -1$ and $y = -\sqrt{3}$ or $x = 1$ and $y = \sqrt{3}$.

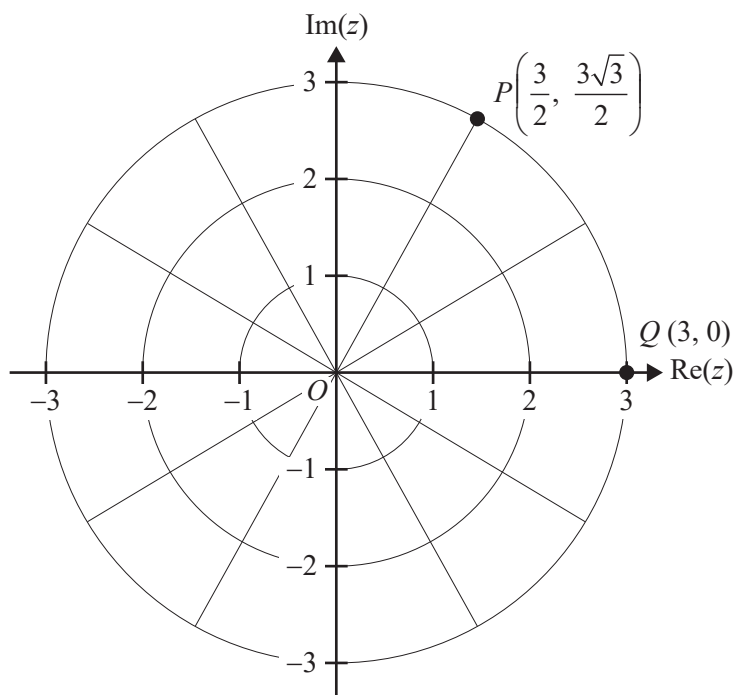
Points of intersection: $-1 - \sqrt{3}i$ and $1 + \sqrt{3}i$.

$\begin{aligned} &\text{solve } (x^2 + y^2 = 4 \text{ and } y = \sqrt{3} \cdot x, x, y) \\ &x = -1 \text{ and } y = -\sqrt{3} \text{ or } x = 1 \text{ and } y = \sqrt{3} \end{aligned}$
--

- ii. Label these points on the Argand plane in **part a**.

1 mark

Consider the points P and Q labelled on the Argand plane below.



- d. A ray originating at point P and passing through point Q has the equation $\text{Arg}(z - z_0) = \theta$, where θ is a radian measure.

Write down the values of z_0 and θ .

1 mark

$$m_{PQ} = -\sqrt{3}; \text{ since } \tan(\theta) = -\sqrt{3}, \text{ then } \theta = -\frac{\pi}{3}$$

The ray $\text{Arg } z = -\frac{\pi}{3}$ starts from O .

The ray starting from P is a translation from O by $\frac{3}{2}$ units right and $\frac{3\sqrt{3}}{2}$ units vertically up.

$$\text{So } z_0 = \frac{3}{2} + \frac{3\sqrt{3}}{2}i.$$

$0 - \frac{3 \cdot \sqrt{3}}{2}$	$-\sqrt{3}$
$3 - \frac{3}{2}$	
$\tan\left(-\frac{\pi}{3}\right)$	$-\sqrt{3}$

- e. Find the area of the minor segment bounded by the chord connecting the points P and Q and the circle given by $|z| = 3$.

Give your answer in the form $c\pi + d$, where $c, d \in \mathbb{R}$.

2 marks

$A = \frac{1}{2} \times 3^2 \times \left(\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right) \right)$	$\frac{1}{2} \cdot 3^2 \cdot \left(\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right) \right)$	$\frac{3 \cdot (2 \cdot \pi - 3 \cdot \sqrt{3})}{4}$
$= \frac{3\pi}{2} - \frac{9\sqrt{3}}{4}$	$\text{propFrac}\left(\frac{3 \cdot (2 \cdot \pi - 3 \cdot \sqrt{3})}{4}\right)$	$\frac{3 \cdot \pi}{2} - \frac{9 \cdot \sqrt{3}}{4}$

Question 3 (10 marks)

A tank initially contains 5 kg of salt dissolved in 3000 litres of water. Salty water that contains 0.1 kg of salt per litre of water enters the tank at a rate of 20 litres per minute. The solution is kept thoroughly mixed and drains from the tank via a tap at the same rate of 20 litres per minute.

- a. By considering concentration, explain whether the quantity of salt in the tank increases with time.

1 mark

The initial concentration $\frac{5}{3000} = \frac{1}{600} = 0.0016\ldots \text{kg/L}$

Incoming salty water concentration = 0.1 kg/L

Since $0.1 > 0.0016\ldots$ the quantity of salt increases with time.

- b. Let Q denote the quantity of salt, in kilograms, in the tank at time t minutes.

Show that Q satisfies the differential equation $\frac{dQ}{dt} = \frac{300-Q}{150}$.

1 mark

Quantity in: 0.1 kg/L at 20 L/min so 2 kg/min

Quantity out: $\frac{Q}{3000} \text{ kg/L}$ at 20 L/min so $\frac{Q}{150} \text{ kg/min}$

$\frac{dQ}{dt} = \text{quantity in} - \text{quantity out}$

$= 2 - \frac{Q}{150} = \frac{300-Q}{150}$

- c. Using Euler's method with a step size of 15 minutes, find $Q(30)$, the approximate quantity of salt in the tank after 30 minutes.

Give your answer in kilograms, correct to two decimal places.

2 marks

$Q_0 = 5(\text{kg})$;

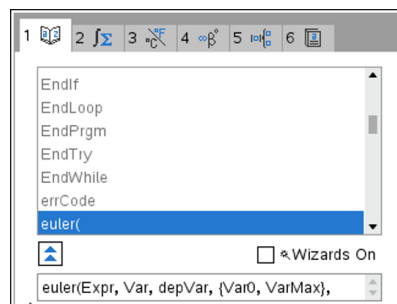
$Q_1 = 34.5(\text{kg})$ (after 15 mins);

$Q_2 = 61.05(\text{kg})$ (after 30 mins)

euler($\frac{300-Q}{150}, t, Q, \{0, 30\}, 5, 15$)

0.	15.	30.
5.	34.5	61.05

Note: Euler command found in Catalog



- d. Use calculus to solve the differential equation $\frac{dQ}{dt} = \frac{300-Q}{150}$, expressing Q in terms of t . 3 marks

$$\int \frac{1}{300-Q} dQ = \int \frac{1}{150} dt$$

$$\frac{t}{150} = -\log_e(300-Q) + c, \quad 0 < Q < 300.$$

$$\text{When } t=0, Q=5, \text{ so } c = \log_e 295$$

$$\frac{t}{150} = \log_e \left(\frac{295}{300-Q} \right) \text{ and so } -\frac{t}{150} = \log_e \left(\frac{300-Q}{295} \right)$$

$$e^{-\frac{t}{150}} = \frac{300-Q}{295} \Rightarrow 300-Q = 295e^{-\frac{t}{150}}$$

$$\text{So } Q = 300 - 295e^{-\frac{t}{150}}$$

$$\text{deSolve}\left(q' = \frac{300-q}{150} \text{ and } q(0)=5, t, q\right)$$

$$q = 300 - 295 \cdot e^{-\frac{t}{150}}$$

- e. What value does the quantity of salt in the tank approach as time approaches infinity?
Give your answer in kilograms.

1 mark

Q is a decreasing function.

As $t \rightarrow \infty$, $e^{-\frac{t}{150}} \rightarrow 0$ and so $Q \rightarrow 300$ (kg)

$$\lim_{t \rightarrow \infty} \left(300 - 295 \cdot e^{-\frac{t}{150}} \right) = 300$$

- f. Find the time taken for the quantity of salt in the tank to reach 100 kg.

1 mark

$$t = 150 \log_e \left(\frac{295}{300-100} \right)$$

$$= 150 \log_e \left(\frac{59}{40} \right) \text{ (min)}$$

$$\text{solve}\left(q = 300 - 295 \cdot e^{-\frac{t}{150}}, t | q = 100\right)$$

$$t = 150 \cdot \ln \left(\frac{59}{40} \right)$$

- g. When the quantity of salt in the tank reaches 100 kg, the tap draining the tank is turned off. Assume that the tank does not overflow and there is no change to the inflow rate.

After the tap is turned off, how many minutes does it take for the concentration of salt in the tank to reach $\frac{1}{20}$ kg L⁻¹?

1 mark

t min after tap is turned off, there will be $(100 + 2t)$ kg in the tank
and the volume will be $(3000 + 20t)$ L.

Concentration of salt after t min is $\frac{100 + 2t}{3000 + 20t}$.

Solving $\frac{100 + 2t}{3000 + 20t} = \frac{1}{20}$ for t gives $t = 50$ (min)

$$\text{solve}\left(\frac{100 + 2 \cdot t}{3000 + 20 \cdot t} = \frac{1}{20}, t\right)$$

$$t = 50$$

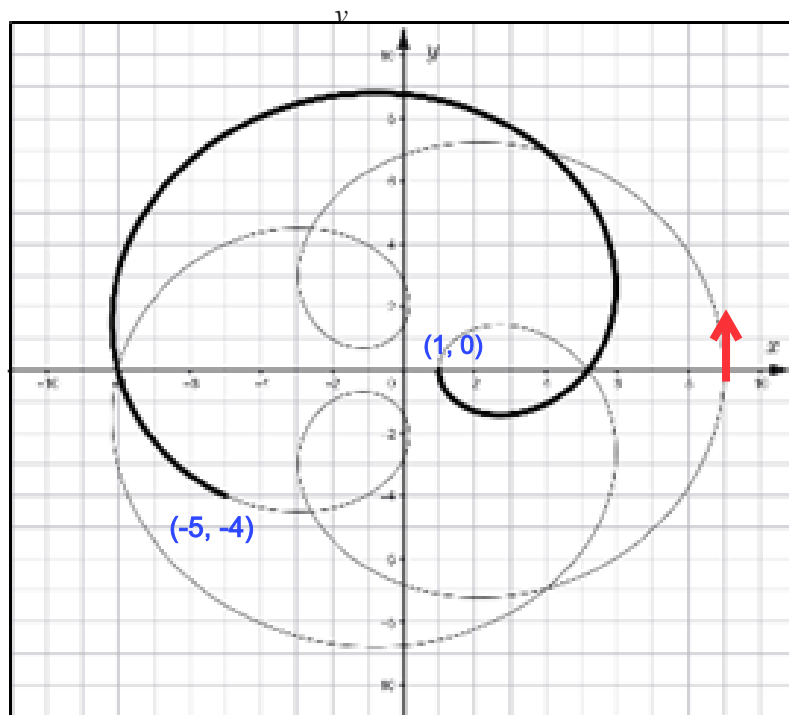
Question 4 (10 marks)

The path of a moving particle with position vector

$$\underline{r}(t) = \left(5 \cos(t) - 4 \cos\left(\frac{5t}{2}\right) \right) \underline{i} + \left(5 \sin(t) - 4 \sin\left(\frac{5t}{2}\right) \right) \underline{j}$$

is shown below for time $t \geq 0$.

All lengths are in **metres** and time is measured in **seconds**.



- a. Write down the **coordinates** of the particle's **starting point**.

1 mark

$\underline{r}(0) = \underline{i}$ and so the starting point is (1, 0)

- b. On the graph above, **draw an arrow from the point (9, 0)** to indicate the direction of motion of the particle.

1 mark

- c. Find the value of t for which the particle will **first return to its starting point**.

1 mark

At $t = 0$, $\cos(t) = \cos\left(\frac{5t}{2}\right) = 1$ and $\sin(t) = \sin\left(\frac{5t}{2}\right) = 0$

Since $\cos(t)$ and $\sin(t)$ have period 2π , they will reach the same values for $t = 0, 2\pi, 4\pi, \dots$

At $t = 2\pi$, $\cos\left(\frac{5t}{2}\right) = -1$ and $\sin\left(\frac{5t}{2}\right) = 0$.

At $t = 4\pi$, $\cos\left(\frac{5t}{2}\right) = 1$ and $\sin\left(\frac{5t}{2}\right) = 0$.

So, the first time where all sin and cos terms reach the same values is at $t = 4\pi$.

$\underline{r}(t) := \left[5 \cdot \cos(t) - 4 \cdot \cos\left(\frac{5 \cdot t}{2}\right) \quad 5 \cdot \sin(t) - 4 \cdot \sin\left(\frac{5 \cdot t}{2}\right) \right]$	
	Done
$\underline{r}(0)$	[1 0]
$\underline{r}(2 \cdot \pi)$	[9 0]
$\underline{r}(4 \cdot \pi)$	[1 0]

- d. Show that the speed of the particle, in m s^{-1} , at time t can be expressed as

$$\sqrt{125 - 100 \cos\left(\frac{3t}{2}\right)}. \quad \text{speed is the magnitude of velocity vector}$$

3 marks

$$\begin{aligned} \dot{\mathbf{r}}(t) &= \left(-5 \sin(t) + 10 \sin\left(\frac{5t}{2}\right)\right) \mathbf{i} + \left(5 \cos(t) - 10 \cos\left(\frac{5t}{2}\right)\right) \mathbf{j} \\ |\dot{\mathbf{r}}(t)|^2 &= \left(-5 \sin(t) + 10 \sin\left(\frac{5t}{2}\right)\right)^2 + \left(5 \cos(t) - 10 \cos\left(\frac{5t}{2}\right)\right)^2 \\ &= 25 \sin^2(t) - 100 \sin(t) \sin\left(\frac{5t}{2}\right) + 100 \sin^2\left(\frac{5t}{2}\right) + 25 \cos^2(t) - 100 \cos(t) \cos\left(\frac{5t}{2}\right) + 100 \cos^2\left(\frac{5t}{2}\right) \\ &= 125 - 100 \left(\sin(t) \sin\left(\frac{5t}{2}\right) + \cos(t) \cos\left(\frac{5t}{2}\right)\right) \\ &= 125 - 100 \cos\left(\frac{5t}{2} - t\right) \\ &= 125 - 100 \cos\left(\frac{3t}{2}\right) \end{aligned}$$

So $|\dot{\mathbf{r}}(t)| = \sqrt{125 - 100 \cos\left(\frac{3t}{2}\right)}$

$$\frac{d}{dt}(\mathbf{r}(t))$$

$$\left[10 \cdot \sin\left(\frac{5 \cdot t}{2}\right) - 5 \cdot \sin(t) \quad 5 \cdot \cos(t) - 10 \cdot \cos\left(\frac{5 \cdot t}{2}\right) \right]$$

- e. What is the maximum speed of the particle in m s^{-1} ?

1 mark

$$|\dot{\mathbf{r}}(t)|_{\max} \text{ occurs when } \cos\left(\frac{3t}{2}\right) = -1$$

$$\text{So } |\dot{\mathbf{r}}(t)|_{\max} = 15$$

- f. On the graph on page 18, trace the path of the particle for $t \in [0, \pi]$.

(Note: endpoints
have been added,
but are not required)

1 mark

- g. Find the length of the path traced in part f, giving your answer in metres, correct to one decimal place.

2 marks

$$L = \int_0^\pi \sqrt{125 - 100 \cos\left(\frac{3t}{2}\right)} dt = 36.6078$$

$$= 36.6 \text{ (m)}$$

$$\int_0^\pi \sqrt{125 - 100 \cdot \cos\left(\frac{3 \cdot t}{2}\right)} dt$$

Question 5 (10 marks)

Consider three planes defined by the equations $\Pi_1: 2x + 9z = 8$, $\Pi_2: 3x + 6y + 5z = 7$ and $\Pi_3: x + 9y - 3z = 7$.

[Menu > Algebra > Solve System of Equations > Solve System of Linear Equations](#)

- a. Find the **point of intersection** of the three planes.

1 mark

Solving the three Cartesian equations simultaneously gives $(-5, 2, 2)$.

$$\text{linSolve}\left(\begin{cases} 2 \cdot x + 9 \cdot z = 8 \\ 3 \cdot x + 6 \cdot y + 5 \cdot z = 7 \\ x + 9 \cdot y - 3 \cdot z = 7 \end{cases}, \{x, y, z\}\right) \\ \{-5, 2, 2\}$$

- b. i. Find a vector that gives the **direction of the line of intersection of the planes Π_2 and Π_3** .

[Menu > Matrix and Vector > Vector > Cross Product](#)

2 marks

$$(3\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}) \times (\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}) = -63\mathbf{i} + 14\mathbf{j} + 21\mathbf{k}$$

$$\text{crossP}([3 \ 6 \ 5], [1 \ 9 \ -3]) \quad [-63 \ 14 \ 21]$$

Alternatively, solving the two cartesian equations gives:

$$x = 1 - 3\lambda, y = \frac{2 + 2\lambda}{3}, z = \lambda \text{ and so any direction vector}$$

$$\text{is a multiple of } -3\mathbf{i} + \frac{2}{3}\mathbf{j} + \mathbf{k}$$

$$\text{linSolve}\left(\begin{cases} 3 \cdot x + 6 \cdot y + 5 \cdot z = 7 \\ x + 9 \cdot y - 3 \cdot z = 7 \end{cases}, \{x, y, z\}\right) \\ \left\{ -(3 \cdot c1 - 1), \frac{2 \cdot (c1 + 1)}{3}, c1 \right\}$$

- ii. Find a **set of parametric equations** that give the coordinates of the points that lie on this line of intersection.

1 mark

The line passes through $(-5, 2, 2)$ and has a direction vector $-63\mathbf{i} + 14\mathbf{j} + 21\mathbf{k}$.

Hence $x = -5 - 63\mu$, $y = 2 + 14\mu$, $z = 2 + 21\mu$.

- c. Find the **shortest distance** from the point $(1, 1, 2)$ to the plane Π_3 .

2 marks

Choose $A(7, 0, 0)$ and $B(1, 1, 2)$.

So $\overrightarrow{AB} = -6\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$$\hat{n} = \frac{1}{\sqrt{91}}(\mathbf{i} + 9\mathbf{j} - 3\mathbf{k})$$

Shortest distance:

$$\begin{aligned} |\overrightarrow{AB} \cdot \hat{n}| &= \left| (-6\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{\sqrt{91}}(\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}) \right| \\ &= \left| -\frac{3}{\sqrt{91}} \right| \\ &= \frac{3}{\sqrt{91}} \end{aligned}$$

Menu > Matrix and Vector > Vector > Dot Product

Menu > Matrix and Vector > Vector > Unit Vector

Use abs instead of norm as the dot product is scalar

$$\left| \text{dotP}\left(\begin{bmatrix} -6 & 1 & 2 \end{bmatrix}, \frac{1}{\sqrt{91}} \cdot \begin{bmatrix} 1 & 9 & -3 \end{bmatrix}\right) \right| = \frac{3 \cdot \sqrt{91}}{91}$$

- d. Consider a family of planes, Ψ , with equation $6x + 27z = m$, where $m \in \mathbb{R}$.

- i. Show that the plane Π_1 is **parallel** to each member of Ψ .

1 mark

A normal to Ψ is $6\mathbf{i} + 27\mathbf{k}$ and a normal to Π_1 is $2\mathbf{i} + 9\mathbf{k}$.

As $6\mathbf{i} + 27\mathbf{k} = 3(2\mathbf{i} + 9\mathbf{k})$, the normals have the same direction and so $\Psi \parallel \Pi_1$.

- ii. Find **all values of** m for which the shortest distance between plane Π_1 and the plane of the form $6x + 27z = m$ is $\frac{23}{3\sqrt{85}}$.

3 marks

Menu > Matrix and Vector > Vector > Dot Product

Choose $A(4, 0, 0)$ and $B\left(\frac{m}{6}, 0, 0\right)$.

So $\overrightarrow{AB} = \left(\frac{m}{6} - 4\right)\mathbf{i}$

$$\hat{n} = \frac{1}{\sqrt{85}}(2\mathbf{i} + 9\mathbf{k})$$

Shortest distance:

$$\begin{aligned} |\overrightarrow{AB} \cdot \hat{n}| &= \left| \left(\left(\frac{m}{6} - 4\right)\mathbf{i}\right) \cdot \frac{1}{\sqrt{85}}(2\mathbf{i} + 9\mathbf{k}) \right| \\ &= \frac{|m - 24|\sqrt{85}}{255} \end{aligned}$$

Solving $\frac{|m - 24|\sqrt{85}}{255} = \frac{23}{3\sqrt{85}}$ for m gives $m = 1, 47$

$$\left| \text{dotP}\left(\begin{bmatrix} \frac{m}{6} - 4 & 0 & 0 \end{bmatrix}, \frac{1}{\sqrt{85}} \cdot \begin{bmatrix} 2 & 0 & 9 \end{bmatrix}\right) \right| = \frac{|m - 24| \cdot \sqrt{85}}{255}$$

$$\text{solve}\left(\frac{|m - 24| \cdot \sqrt{85}}{255} = \frac{23}{3 \cdot \sqrt{85}}, m\right)$$

$m = 1 \text{ or } m = 47$

Question 6 (10 marks)

The volume of water, V mL, consumed by a student during a school day may be assumed to be normally distributed with a mean of 1000 mL and a standard deviation of 80 mL.

- a. i. Write down the mean and standard deviation of the sampling distribution for the average volume of water consumed by randomly selected samples of 25 students.

Give your answers in millilitres.

1 mark

$$E(\bar{V}) = 1000 \text{ and } sd(\bar{V}) = \frac{80}{\sqrt{25}} = 16$$

- ii. What is the probability, correct to four decimal places, that the average volume of water consumed by a random sample of 25 students on a particular school day is more than 970 mL?

1 mark

$$\bar{V} \sim N(1000, 16^2)$$

Menu > Probability > Distributions > Normal Cdf

$$\Pr(\bar{V} > 970) = 0.9696$$

normCdf(970, ∞, 1000, 16)	0.969604
---------------------------	----------

The canteen at a particular school stocks two brands of water in bottles, Wasser and Apa.

The manufacturer of Wasser bottled water knows that the volume of water dispensed into bottles may be assumed to be normally distributed with a standard deviation of 5 mL.

Engineers at the company take a random sample of 30 bottles and measure the volume of water in each bottle. The sample mean is found to be 750 mL.

- b. Find a 95% confidence interval for the mean volume of water dispensed into each Wasser bottle.

Give your values in millilitres, correct to one decimal place.

Menu > Statistics > Confidence Intervals > z Interval ...

$$(748.2, 751.8)$$

zInterval 5,750,30,0.95: stat.results	
"Title"	"z Interval"
"CLower"	748.211
"CUpper"	751.789
"x"	750.
"ME"	1.78919
"n"	30.
"σ"	5.

1 mark

- c. The engineers decide to take 300 random samples, each containing 30 bottles, and calculate the respective 95% confidence intervals. All samples are independent.

In how many of these confidence intervals would the engineers expect the value of the true mean volume dispensed to be included?

1 mark

$$0.95 \times 300 = 285$$

0.95 · 300	285.
------------	------

- d. What is the minimum size of the sample required to ensure that the difference between the sample mean and the mean volume dispensed is no more than 1 mL at the 95% confidence level?

1 mark

Menu > Probability > Distributions > Inverse Normal ...

$$\text{Solving } 1.95996... \times \frac{5}{\sqrt{n}} \leq 1 \text{ for } n \text{ gives } n \geq 96.03...$$

The minimum sample size is 97

solve(invNorm(0.975, 0, 1) · $\frac{5}{\sqrt{n}} \leq 1, n$)	$n \geq 96.0365$
---	------------------

The volume of water dispensed into Apa water bottles may be assumed to be normally distributed with a mean of 750 mL and a standard deviation of 5 mL. After a service, a random sample of 50 bottles gave a sample mean of 748 mL. The company now claims that the mean volume of water dispensed is less than the stated mean of 750 mL.

A one-tailed statistical test at the 1% level of significance is proposed.

- e. Write down the null and alternative hypotheses that will be used in testing the company's claim.

1 mark

$$H_0: \mu = 750 \text{ and } H_1: \mu < 750$$

- f. i. Determine the p -value for this test.

Give your answer correct to four decimal places.

1 mark

$$\text{If } H_0 \text{ is true then } \bar{V} \sim N\left(750, \left(\frac{5}{\sqrt{50}}\right)^2\right)$$

$$p = \Pr(\bar{V} < 748 | H_0) = 0.0023$$

$$\text{normCdf}\left(-\infty, 748, 750, \frac{5}{\sqrt{50}}\right) \quad 0.002339$$

- ii. Is the company's claim correct?

Explain your conclusion in terms of the p value.

1 mark

Since $0.0023 < 0.01$, there is strong evidence to reject H_0 and accept H_1 that the company's claim that the mean is less than 750 mL.

- g. At the 1% level of significance for a sample size of 50 bottles, find the critical value of the sample mean, below which a sample mean value would support the conclusion that the mean volume of water dispensed is now less than 750 mL.

Give your answer correct to three decimal places.

1 mark

$$\Pr(\bar{V} < c) \leq 0.01$$

$$c = 748.355(\text{ml})$$

$$c := \text{invNorm}\left(0.01, 750, \frac{5}{\sqrt{50}}\right) \quad 748.355$$

- h. Assume that, after the service, the true mean volume of water in the Apa bottles was found to be 747.5 mL and that the population standard deviation, σ , is 5 mL.

At the 1% level of significance, for a sample size of 50, find the probability that the company will conclude that the service has not reduced the mean volume of water in an Apa bottle.

Give your answer correct to three decimal places.

1 mark

$$\Pr(\bar{V} > 748.355 | \mu = 747.5) = 0.113$$

$$\text{normCdf}\left(c, \infty, 747.5, \frac{5}{\sqrt{50}}\right) \quad 0.113296$$