Infinite Geometric Series

ID: 11065

Time required
45 minutes

Activity Overview

In this activity, students will explore infinite geometric series. They will consider the effect of the value for the common ratio and determine whether an infinite geometric series converges or diverges. They will also consider the derivation of the sum of a convergent infinite geometric series and use it to solve several problems.

Topic: Sequences & Series

- *Explore geometric sequences*
- *Sum a geometric series*
- *Convergence of an infinite geometric series*

Teacher Preparation and Notes

- *This activity serves as an introduction to infinite geometric series. Students will need to have previously learned about finite geometric series.*
- *Before beginning the activity, students need to clear all lists and turn off all plots and equations. To clear all lists, press* 2nd [MEM] PRGM*, scroll down to* ***ClrAllLists*** *and press* ENTER*.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "11065" in the keyword search box.***

Associated Materials

- *InfiniteGeometricSeries_Student.doc*

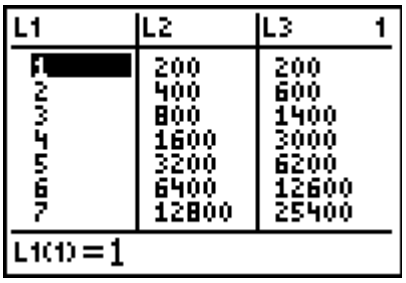
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Geometric Sequences & Series (TI-84 Plus family) — 8682*
- *Spreading Doom (TI-84 Plus family) — 10073*

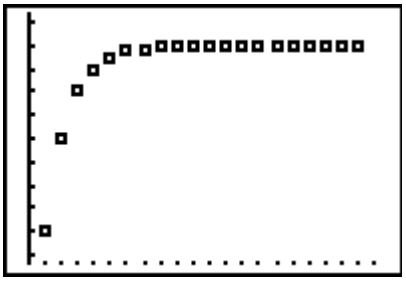
Problem 1 – Investigating Infinite Geometric Series

Students will explore what happens when the common ratio changes for an infinite geometric series. For each value of *r*, students will create lists L2 and L3 and then scroll through L3 to determine if the series converges or diverges.



As an extension, students could change the initial value of the sequence by changing the number 200 in the formula for L2 and see if it affects the convergence or divergence of the series.

If students determine that a series converges, then they are to create and view the scatter plot. The necessary settings for **Plot1** are shown on the student worksheet.



To change the window, students can press ZOOM and select **ZoomStat** or manually adjust it by pressing WINDOW.

1.

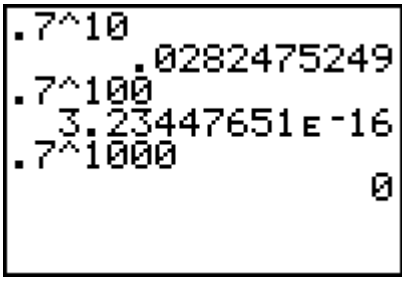
r	–2	–0.5	–0.25	0.25	0.5	2
Converges or Diverges	Diverges	Converges 133.33	Converges 160	Converges 266.667	Converges 400	Diverges

2. $|r| < 1$

3. There is a horizontal asymptote at the point of convergence.

Problem 2 – Deriving a Formula for the Sum of a Convergent Infinite Geometric Series

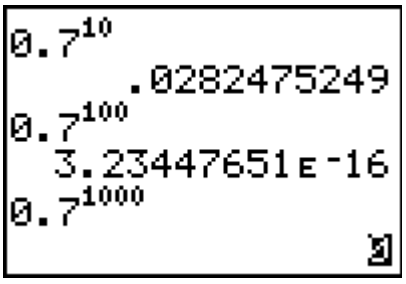
Students are to use the Home screen to determine the values of r^n when $r = 0.7$. They should see that as n gets very large r^n becomes zero when $|r| < 1$.



Students are given the formula for the sum of a *finite* geometric series. With the information found on the worksheet, they can determine the formula for the sum of an *infinite* geometric series using substitution.

If using Mathprint OS:

When students press the ^ key, the cursor will move up to the exponent. Students can enter the exponent, 10, 100, or 1000, and then press the right arrow key to move out of the exponent.



4. Note that even though the calculator says r^{1000} and r^{10000} both equal zero, they are actually very small numbers and are *approximately* zero.

n	10	100	1000	10000
$r^n = 0.7^n$	0.028248	3.23E–16	0	0

5. $S_n = \dfrac{a_1(1-0)}{1-r} = \dfrac{a_1}{1-r}$

Problem 3 – Apply what was learned

In this problem, students are given a scenario relating to drug prescriptions and dosages. Students need to use the formulas shown in the previous problem to answer the questions.

They may get caught up on the first question. Explain to students that if 15% of the drug leaves the body every hour, then that means that 85% percent is still in the body.

6.

a. 0.40 = 1 – 0.15(4)

b. 240 + 240(0.4) = 336 mg

c.

Hours	0 (1st dosage)	4 (2nd dosage)	8 (3rd dosage)	12	16
Amount in the Body	240	336	374.4	389.76	395.904

d. This is the 7th dosage, so $S_7 = \dfrac{240(1-.4^7)}{1-0.4} = 399.34464$.

e. This is the 19th dosage, so $S_{19} = \dfrac{240(1-.4^{19})}{1-0.4} = 399.999989$.

f. $S_t = \dfrac{240(1-.4^t)}{1-0.4}$

g. No, since $S = \dfrac{240}{1-0.4} = 400$, you will not reach the minimum lethal dosage.

f. Yes, since he/she waits 2 hours, only 30% of the drug is out of his/her system, so 70% remains. This is the common ratio $r = 0.7$ and $S = \dfrac{240}{1-0.7} = 800$