Jack's Tile Problem – Exploring the Area Rule for Triangles. Teacher Answers

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TI-Nspire CAS **Investigation** **Student** **50 min**

Aim

The aim of this investigation is to use various triangles to explore, develop and apply the area rule.

Equipment

For this activity you will need:

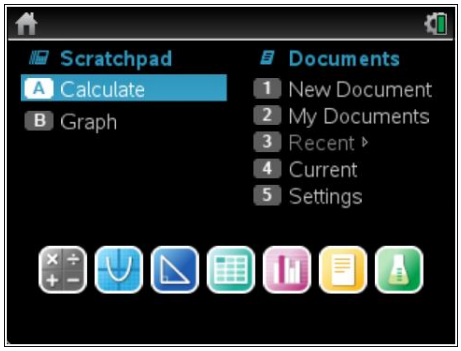
- TI-Nspire CAS
- TI-Nspire CAS file: Trig Area Rule

Problem Description

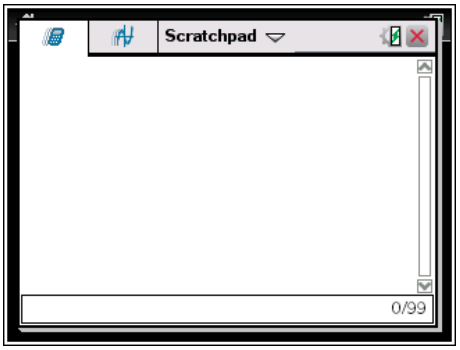
The rules that you would have studied to date (Area = $\frac{1}{2}$ × base × height, Pythagoras' Theorem and SOHCAHTOA) are useful for calculating unknown information about right-angled triangles and triangles given a perpendicular height. Unfortunately, not all situations involve such triangles. How can we find the area of triangles that are not right-angled or do not give a base and height measurement?

Technology

The activity requires access to the "Trig Area Rule" TI-Nspire document. This document should be loaded on your device before proceeding. You should also ensure your calculator mode is set to **Degree**.



Part of in this investigation requires calculations to be performed. The Scratchpad is a place where calculations can be computed and then discarded. To access the Scratchpad press **home** and select **Scratchpad** (or press **A**). Alternatively, press the [doc] key (this key is not available on a Clickpad).



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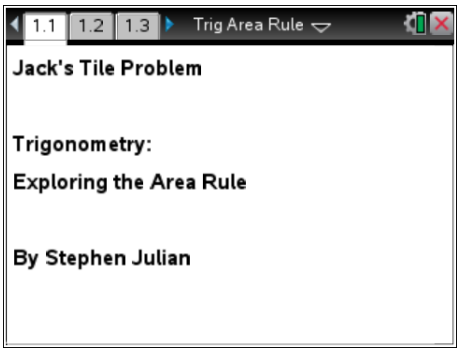
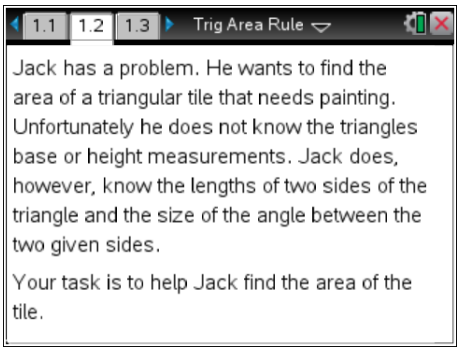
Open the "Trig Area Rule" file. Page **1.1** is the introductory page.



Explanation of the Problem

Read pages **1.2** to **1.6** to outline the problem you are about to explore.

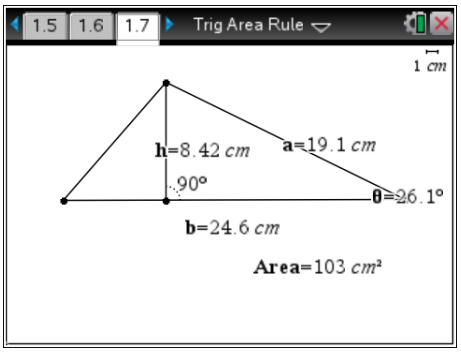


Manipulation of the Triangle and Recording of Results

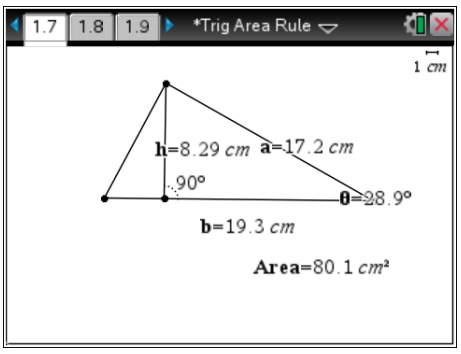
Navigate to page **1.7**. Grab and move vertices of the triangle to form different triangles.

Once you are happy with a triangle, you can record the results into the table on page **1.8** by pressing **[ctrl] + [.]**.

This action will capture the base, height and area measurements for a given triangle.



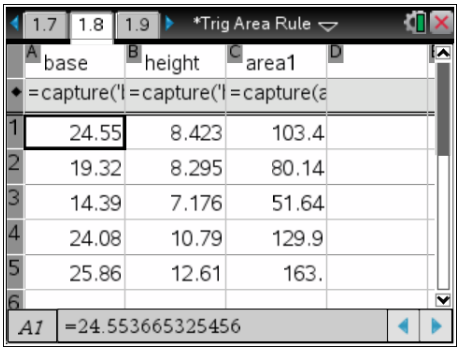
Repeat this process four more times (ensure the height measurement is always visible or it will not be recorded). You will now have captured the base, height and area measurements for five different triangles.



Viewing the Results

Navigate to page **1.8** to view the results of the data capture. If you decide to delete and then recapture a set of measurements, highlight and delete a row of data. When prompted to overwrite the data, select 'Yes'. Navigate to page **1.7** and capture any new data.

When you have finished collecting data for five different triangles, record your results in the table for Question 1.



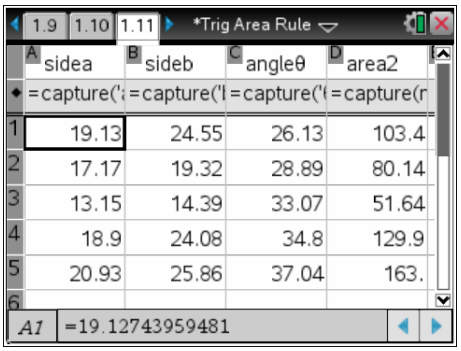
Looking at the Area Rule

Read pages **1.9** and **1.10** to learn about the area rule for a triangle given two sides and the included angle.

Page **1.11** shows the measurements for the five triangles you created. However this time the area has been calculated using the formula Area $= \frac{1}{2}ab\sin(\theta)$ (delete any extra rows of measurements you may have created).

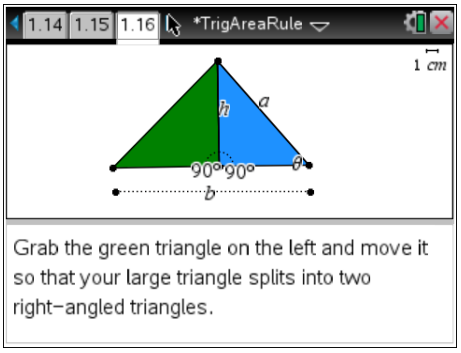
Copy these results to the table in Question 3.

Complete Questions 4, 5 and 6 below.

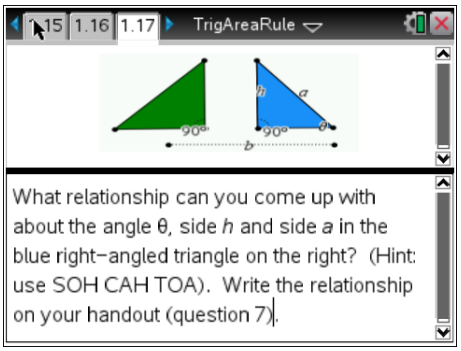


Proof of the Area Rule

Read pages **1.14** to **1.18** to see a "Proof of the Area Rule". The triangle can be separated by grabbing the green triangle and moving it to the left.



After reading these pages and moving the triangles, complete Questions 7, 8 and 9 below to see if you understand how this area rule works.



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Questions

1. Record your results from page **1.8** on your TI-Nspire in the table below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS.

Triangle	Base	Height	Area
1	24.55	8.423	103.4
2	19.32	8.295	80.14
3	14.39	7.176	51.64
4	24.08	10.79	129.9
5	25.86	12.61	163

2. Use the base and height measurement from the table in question 1 and the formula

$$\textbf{Area} = \frac{1}{2}\ \textbf{base x height}$$

to check the area values that were generated on your TI-Nspire CAS. Use the Scratchpad for your calculations. Clearly show your working for each triangle in the spaces provided below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS.

Triangle 1:

Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 24.55 \times 8.432$

$= 103.503\ cm^2$

Triangle 2:

Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 19.32 \times 8.295$

$= 80.1297\ cm^2$

Triangle 3:

Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 14.39 \times 7.176$

$= 51.6313\ cm^2$

Triangle 4:

Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 24.08 \times 10.79$

$= 129.912\ cm^2$

Triangle 5:

Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 25.86 \times 12.61$

$= 163.047\ cm^2$

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3. Record your results from page **1.11** on your TI-Nspire CAS in the table below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS.

Triangle	Side A	Side B	Angle θ	Area
1	19.13	24.55	26.13	103.4
2	17.17	19.32	28.89	80.14
3	13.15	14.39	33.07	51.64
4	18.9	24.08	34.8	129.9
5	20.93	25.86	37.04	163

4. Use the *side a*, *side b* and *angle* θ values from the table in question 3 and the formula

$$\textbf{Area} = \frac{1}{2} \textbf{ x } a \textbf{ x } b \textbf{ x } \sin(\theta)$$

to check the area values that were generated on your TI-Nspire CAS. Use the Scratchpad for your calculations (ensure your calculator is set to **Degree** mode).

Clearly show your working for each triangle in the spaces provided below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS.

Triangle 1:

Area = $\frac{1}{2}$ x a x b x $\sin(\theta)$

= $\frac{1}{2}$x19.13x24.55xsin(26.13)

= 103.417 cm^2

Triangle 2:

Area = $\frac{1}{2}$ x a x b x $\sin(\theta)$

= $\frac{1}{2}$x17.17x19.32xsin(28.29)

= 80.1329 cm^2

Triangle 3:

Area = $\frac{1}{2}$ x a x b x $\sin(\theta)$

= $\frac{1}{2}$x13.15x14.39xsin(33.07)

= 51.6275 cm^2

Triangle 4:

Area = $\frac{1}{2}$ x a x b x $\sin(\theta)$

= $\frac{1}{2}$x18.9x24.08xsin(34.8)

= 129.869 cm^2

Triangle 5:

Area = $\frac{1}{2}$ x a x b x $\sin(\theta)$

= $\frac{1}{2}$x20.93x35.86xsin(37.04)

= 163.017 cm^2

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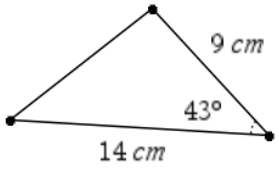
5. Using Area = $\frac{1}{2}$ x a x b x sin(θ) calculate the area of the triangles below, showing full working out and giving your answers correct to two decimal places where necessary.

(a)

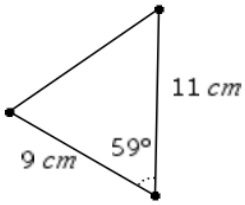


Area = $\frac{1}{2}$ x a x b x sin(θ)

= $\frac{1}{2}$ x 14 x 9 x sin(43)

= 42.97 cm^2

(b)

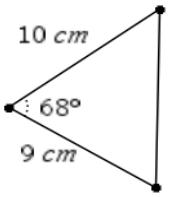


Area = $\frac{1}{2}$ x a x b x sin(θ)

= $\frac{1}{2}$ x 11 x 9 x sin(59)

= 42.43 cm^2

(c)

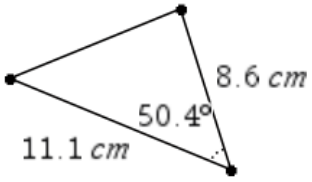


Area = $\frac{1}{2}$ x a x b x sin(θ)

= $\frac{1}{2}$ x 10 x 9 x sin(68)

= 41.72 cm^2

(d)

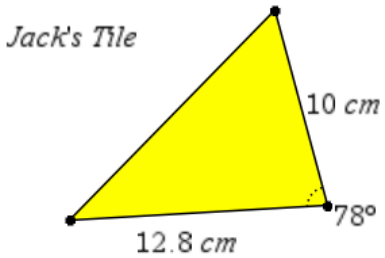


Area = $\frac{1}{2}$ x a x b x sin(θ)

= $\frac{1}{2}$ x 11.1 x 8.6 x sin(50.4)

= 36.78 cm^2

6. Help Jack find the area of his triangular tile.



Area = $\frac{1}{2}$ x a x b x sin(θ)

= $\frac{1}{2}$ x 12.8 x 10 x sin(78)

= 62.60 cm^2

Author: Stephen Julian

7. What is the relationship between the angle θ, side *h* and side *a* in the blue triangle on page **1.17** of your TI-Nspire CAS? (Hint: use SOHCAHTOA)

The relationship is $\sin(\theta) = \frac{h}{a}$

8. Rearrange your relationship in question 7 to make *h* the subject.

$h = a \sin(\theta)$

9. It is known that the area of a triangle can be found using Area = $\frac{1}{2}$ x *b* x *h*.
Substitute your relationship from question 8 where *h* was the subject into Area = $\frac{1}{2}$ x *b* x *h*.
What rule do you get?

Area = $\frac{1}{2}$ x *a* x *b* x $\sin(\theta)$

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