

Simulating Fairness: Exploring Probability

In this lesson, you will create a simulation for the game “Evens or Odds”. Your simulation will allow you to investigate the fairness of different variations of the game. Along the way, you will learn how to use Python to generate random numbers, store values in a list, use a loop to repeat code, and plot data on a scatterplot. You will use experimental probability to make predictions. You’ll use simulations to demonstrate the Law of Large Numbers. Lastly, you will use your simulation and tree diagrams to answer the question, is “Evens or Odds” a fair game under different conditions.

Objectives:

Programming Objectives:

- Use lists to store data
- Use the randint() function to generate random integers.
- Use the plot library to plot points
- Use loops to repeat calculations
- Use if statements to make selections

Math Objectives:

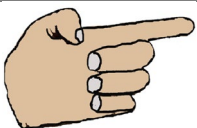



- Use simulations and the Law of Large numbers to investigate probability
- Create and use Tree Diagrams to solve problems.

Math Course Connections: This activity is recommended for Pre-Algebra or Statistics.

In this project, you will synthesize the game “Evens and Odds” and learn some probability along the way. Traditionally, the game is played by two players. One player is “even”. The other play is “odd”. On the count of three each player holds out either one or two fingers. The sum between the two players determines the winner, “evens” or “odds”. Is this a fair game? Does it matter if you pick “evens” or “odds”?

Let’s take a look at the game itself.

Player 1 wins if the sum is even, player 2 wins if the sum is odd. Here a few sample plays of the game.

| Round | Player 1 Plays: | Player 2 Plays: | Result |
|-------|---|---|---------------------------------|
| 1 |  |  | Sum: 2 Player 1 (evens) wins |
| 2 |  |  | Sum: 3 Player 2 (odds) wins |

1. If possible, try the game out for yourself. Play the game 6 times with a partner. Record the number of even wins and the number of odd wins.

Evens:

Odds:

2. Does the game appear to be fair? If not, which should you pick, “Evens” or “Odds”? Why?

Teacher Tip:

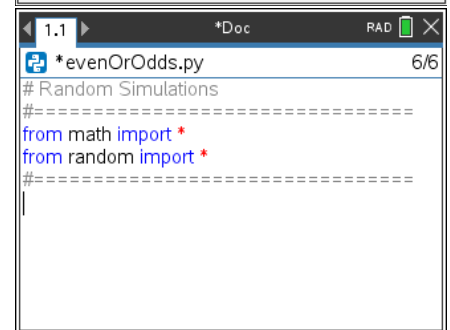
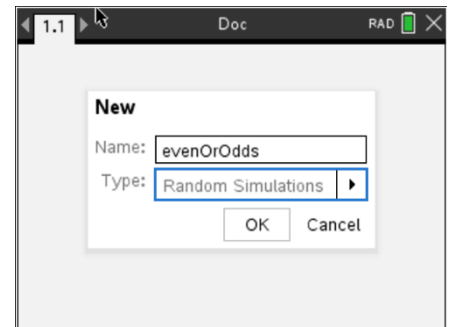
Partner pairs could share out who won overall in each group (even or odd).
Class data could be collected to determine theoretical probability with more samples.

Simulations are often useful to determine if a game is fair. Simulations let you synthesize thousands of games in a short amount of time. In this project, you will create a coding simulation that will let you investigate the probability of “evens” winning after hundreds of games.

1. The first step will be to create a Python random simulation document.

Create a new python project named “evenOrOdds”.

Select “Random Simulations” from the type menu.
This will automatically import the random library.
You will need the randint function from this library.

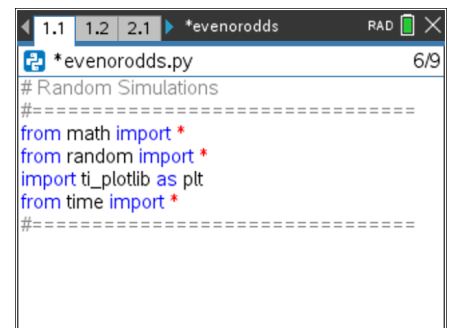


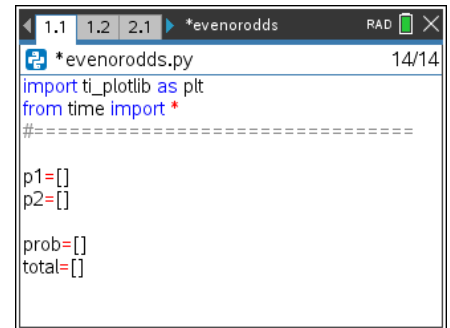
ti_plotlib

Place your cursor on the line below the from random import *
Menu> TIPlotLib > **import ti_plotlib as plt**
This library provides the plotting functions.

time

Place your cursor on the line below the from ti_plotlib import *
Menu> More Modules > Time > **time import ***
This library provides the sleep function.





```

1.1 1.2 2.1 *evenorodds RAD 14/14
import tiplotlib as plt
from time import *
#-----
p1=[]
p2=[]
prob=[]
total=[]

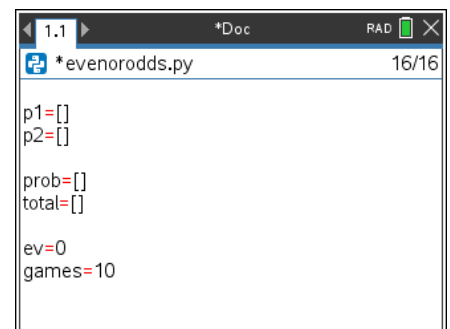
```

4. Create a variable “ev” to keep track of the total number of evens.
Create a variable games to control the number of games.
Set ev equal to 0 and games equal to 10.

```

ev = 0
games = 10

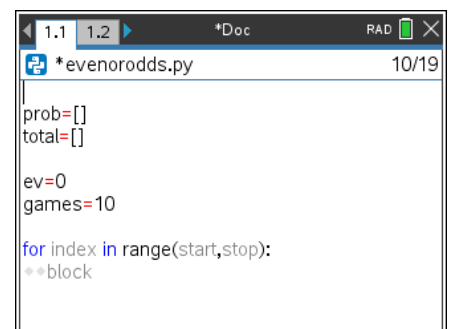
```



```

1.1 *Doc RAD 16/16
p1=[]
p2=[]
prob=[]
total=[]
ev=0
games=10

```



```

1.1 1.2 *Doc RAD 10/19
prob=[]
total=[]
ev=0
games=10
for index in range(start,stop):
    **block

```

6. Replace the place holder “index” with a variable named t for trial.
Let the range start at 1 and stop at games+1.

It might seem a bit strange to say stop at “games+1” when we want 10 rounds. In Python, the statement **for t in range(1,games+1)** will happen 10 times. Initially, t = 1 will start the loop. Each time through the loop, t increases by 1, then checks to make sure the value is less than games+1 before executing the loop.



```

1.1 1.2 *Doc RAD 18/19
prob=[]
total=[]
ev=0
games=10
for t in range(1,games+1):
    **block

```

7. Now to generate and temporarily store each player's random number from 1 to 2. The function randint lets you generate integers.

Add the lines:

```
n1 = randint(1,2)
n2 = randint(1,2)
```

randint:

Menu > Random > randint

Make sure these lines are indented two spaces. You should see two diamonds, `◇◇`, in front of each line. Python uses indentation to keep the lines of a loop grouped together. If the indentation isn't correct, you'll have an error when you run your program.

8. It is a good idea to frequently run your program to look for and fix any errors. It is easier to debug often to avoid several bugs from happening at once.

Add a print statement:

```
print(n1, n2)
```

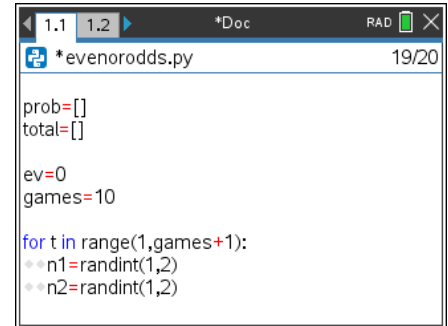
9. Now to find the sum and determine if it is even or odd. If the sum is "even", add 1 to the sum total and display even, otherwise display "odd".

Delete the print statement. Replace it with an if..else statement.

Menu > Built-ins > Control > if..else

Simulating Fairness

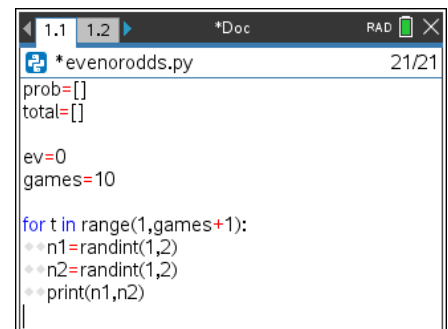
TEACHER NOTES



```
1.1 1.2 *Doc RAD 19/20
*evenorodds.py
prob=[]
total=[]

ev=0
games=10

for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
```

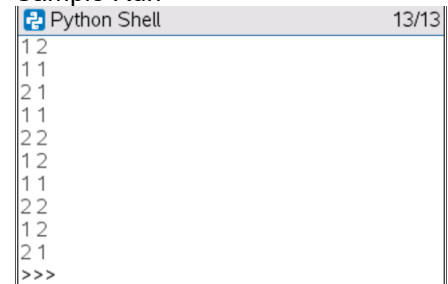


```
1.1 1.2 *Doc RAD 21/21
*evenorodds.py
prob=[]
total=[]

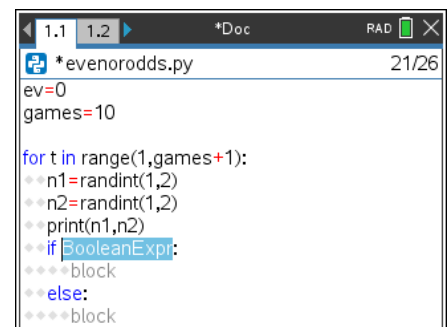
ev=0
games=10

for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
    print(n1,n2)
```

Sample Run



```
Python Shell 13/13
1 2
1 1
2 1
1 1
2 2
1 2
1 1
2 2
1 2
2 1
>>>
```



```
1.1 1.2 *Doc RAD 21/26
*evenorodds.py
ev=0
games=10

for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
    print(n1,n2)
    if BooleanExpr:
        block
    else:
        block
```

Teacher Tip:

Indentation is a common error. If the code doesn't compile or work correctly, check to make sure the indentation matches the code above.

10. To determine if a number is even or odd, use the modulo symbol %.

The modulo symbol determines the remainder after division.


$$1\%2 = 1 \quad 3\%2 = 1 \quad 5\%2 = 1$$

while

$$2\%2 = 0 \quad 4\%2 = 0 \quad 6\%2 = 0$$

Thus, if $(n1 + n2) \% 2 == 0$, the sum is even.

Add this expression to your if statement.

The % can be found in the  button.



```
*evenorodds.py 21/26
ev=0
games=10

for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
    print(n1,n2)
    if (n1+n2)%2==0:
        block
    else:
        block
```

Teacher Tip:

Make sure students use an == in the if statement.

Python uses == to check equivalency. If students use the = sign, it will not check equivalency.

11. If the sum is even, add 1 to the ev counter variable and print “even”, otherwise print “odd”.

Notice on the example to the right the two lines below are indented two more spaces.

```
ev+=1
print(n1,n2, "even")
```

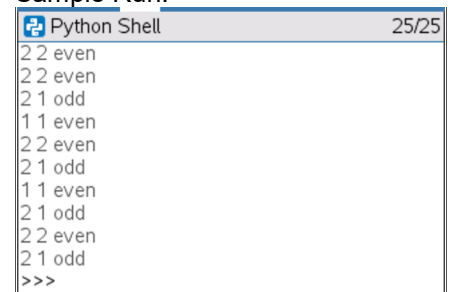
If statements use the same indentation rule as the for loops. Anything that is part of the if should be indented two spaces.



```
*evenorodds.py 25/27
games=10

for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
    print(n1,n2)
    if (n1+n2)%2==0:
        ev+=1
        print(n1,n2,"even")
    else:
        print(n1,n2,"odd")
```

Sample Run:



```
Python Shell 25/25
2 2 even
2 2 even
2 1 odd
1 1 even
2 2 even
2 1 odd
1 1 even
2 1 odd
2 2 even
2 1 odd
>>>
```

12. Now, add the player’s numbers to their lists.

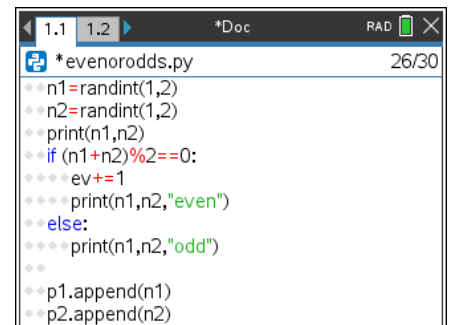
The function **.append** adds to a list.

Unindent the next line of code so it only has only two spaces.

The append is part of the loop but not part of the if statement.

Add the lines:

```
p1.append(n1)
```



```
*evenorodds.py 26/30
n1=randint(1,2)
n2=randint(1,2)
print(n1,n2)
if (n1+n2)%2==0:
    ev+=1
    print(n1,n2,"even")
else:
    print(n1,n2,"odd")
p1.append(n1)
p2.append(n2)
```

p2.append(n2)

.append

Menu > Built-ins > Lists > append

13. Now to calculate the current probability of “evens” winning the game.

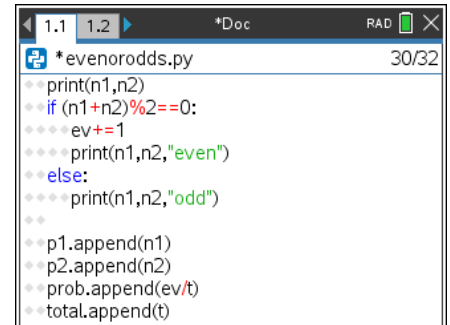
To calculate the probability: $\frac{\text{number of even wins}}{\text{total number of games played}} = \frac{ev}{t}$

Add this probability to the list of probabilities.

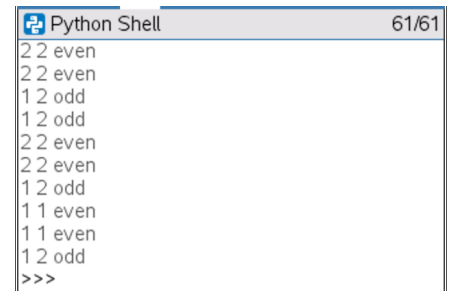
prob.append(ev/t)

Add the total number of games played to the list of totals.

total.append(t)



```
*evenorodds.py 30/32
print(n1,n2)
if (n1+n2)%2==0:
    ev+=1
    print(n1,n2,"even")
else:
    print(n1,n2,"odd")
p1.append(n1)
p2.append(n2)
prob.append(ev/t)
total.append(t)
```

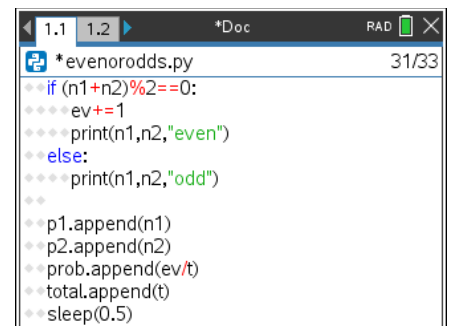


```
Python Shell 61/61
2 2 even
2 2 even
1 2 odd
1 2 odd
2 2 even
2 2 even
1 2 odd
1 1 even
1 1 even
1 2 odd
>>>
```

14. Those numbers scroll by awfully fast! Add the line sleep(.5) to the end of your loop code. This will make the program pause for 0.5 seconds in-between each roll.

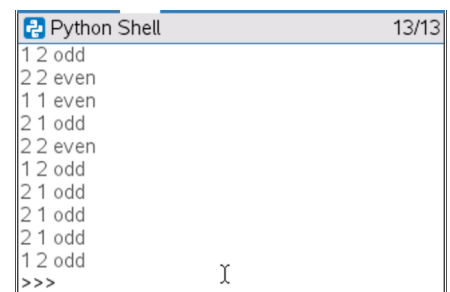
sleep

Menu > More Modules > Time > sleep



```
*evenorodds.py 31/33
if (n1+n2)%2==0:
    ev+=1
    print(n1,n2,"even")
else:
    print(n1,n2,"odd")
p1.append(n1)
p2.append(n2)
prob.append(ev/t)
total.append(t)
sleep(0.5)
```

Run your program. *Do you notice the time difference?*



```
Python Shell 13/13
1 2 odd
2 2 even
1 1 even
2 1 odd
2 2 even
1 2 odd
2 1 odd
2 1 odd
2 1 odd
2 1 odd
1 2 odd
>>>
```

15. Add a print statement after the loop.
Make sure to unindent this print statement so it only happens once.

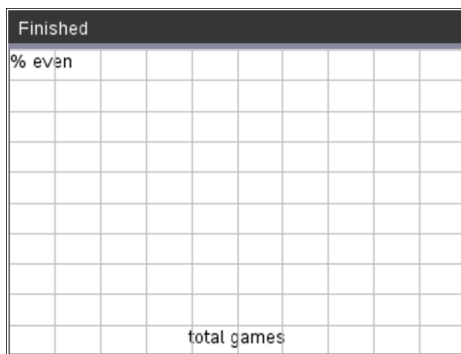
```
print(ev, "/", games, "=", prob[-1])
```



```
*evenorodds.py 33/35
print(n1,n2,"even")
else:
print(n1,n2,"odd")
p1.append(n1)
p2.append(n2)
prob.append(ev/t)
total.append(t)
sleep(0.5)

print(ev,"/",games,"=",prob[-1])
```

16. Now to set up the graphing window.



Question 1:

There are a total of 10 games. What do you think we should set the minimum and maximum values for the x-axis?

min x = min y =

max x = max y =

Question 2:

What is the lowest possible y value for the prob("even")?

Question 3:

What is the highest possible y value for the prob("even")?

Teacher Tip:

- Q1: minx = 0 or 1, max x = 10 or 11 min y = 0, max y = 1
Q2: 0 (zero even wins out of ten)
Q3: 1 (ten even wins out of ten)

17. To set up the window, you need the window function from the plot library.

Menu > TI PlotLib > setup > window

Make the x values go from 0 to games and the y values go from 0 to 1.

```

1.1 1.2 *Doc RAD 32/36
*evenorodds.py
+++ print(n1,n2,"odd")
+++
+++ p1.append(n1)
+++ p2.append(n2)
+++ prob.append(ev/t)
+++ total.append(t)
+++ sleep(0.5)

print(ev,"/",games,"=",prob[-1])

plt.window(0,games,0,1)

```

18. Now add a label to the x and y axis. The generic function is

Menu > TI PlotLib > Draw > text_at

```
plt.text_at(row,"text","align")
```

You will label the y-axis "% even". The y-axis is on the left of the screen.

Therefore, add the line

```
plt.text_at(1,"% even","left")
```

You will label the x-axis "total games". The x-axis label will be in the center.

Therefore, add the line

```
plt.text_at(13,"total games","center")
```

```

1.1 1.2 *Doc RAD 35/38
evenorodds.py
+++ p2.append(n2)
+++ prob.append(ev/t)
+++ total.append(t)
+++ sleep(0.5)

print(ev,"/",games,"=",prob[-1])

plt.window(0,games,0,1)
plt.text_at(1,"% even","left")
plt.text_at(13,"total games","center")

```

19. Plot the data.

Menu > TI PlotLib > Draw > Scatter

The x-list is the total list. Your y-list is prob.

```
plt.scatter(total, prob, "o")
```

```

1.1 1.2 *Doc RAD 38/38
*evenorodds.py
+++ p2.append(n2)
+++ prob.append(ev/t)
+++ total.append(t)
+++ sleep(0.5)

print(ev,"/",games,"=",prob[-1])

plt.window(0,games,0,1)
plt.text_at(1,"% even","left")
plt.text_at(13,"total games","center")
plt.scatter(total,prob,"o")

```

20. Your graph is missing some grid lines.

Menu > TI PlotLib > Setup > grid

Since the x's have a range of [0,10], the x step should be 1.

The y's have a range of [0,1]. Make the y step 0.1

```
plt.grid(games/10,0.1,"solid")
```

```

1.1 1.2 *Doc RAD 32/39
*evenorodds.py
+++ prob.append(ev/t)
+++ total.append(t)
+++ sleep(0.5)

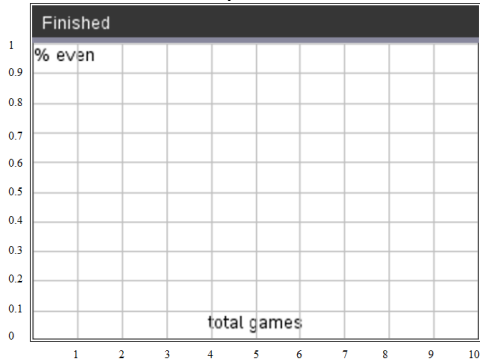
print(ev,"/",games,"=",prob[-1])

plt.window(0,games,0,1)
plt.text_at(1,"% even","left")
plt.text_at(13,"total games","center")
plt.scatter(total,prob,"o")
plt.grid(games/10,0.1,"solid")

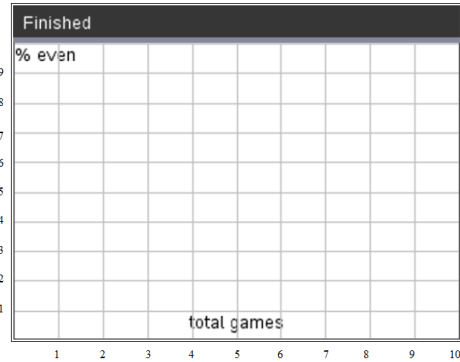
```



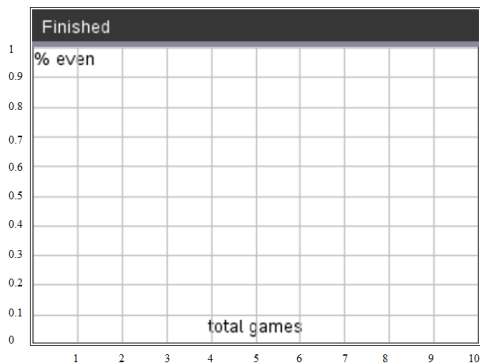

21. Run your program 4 times. Each time you execute your program, copy the graph onto a new graph below. Record the overall percent of evens as well.



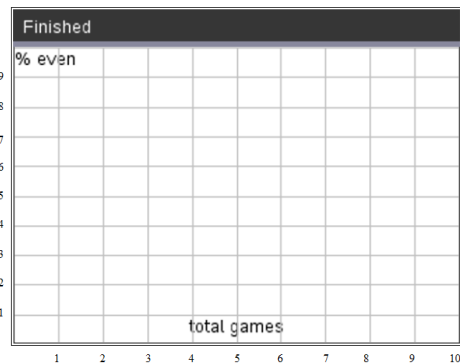
_____ ≈
10



_____ ≈
10



_____ ≈
10



_____ ≈
10

Do you notice any patterns to your data?

According to your graphs, is the game fair? If not, which should you pick, evens or odds?

What do you think would happen if you played 500 games instead of 10?

What would the graphs look like?

What proportion of games would evens win?

22. Let's change your code to run 500 times.

First, let's remove the print statements and the sleep(0.5).

If we leave them in, you'll have to wait $500 \times 0.5 = 250$ seconds.

That's more than 4 MINUTES!

In front of the two **print** statements add a # symbol.

This makes those two lines comments. The computer will skip them.

Add a # to the else statement as well.

Finally, put a # in front of the sleep command.

To add the # symbol, put the cursor at the beginning of a line.

Menu > Edit > Comment

23. Next, change the games variable to equal 500 instead of 10.

```
*evenorodds.py 15/39
p2=[]
prob=[]
total=[]
ev=0
games=500
for t in range(1,games+1):
    n1=randint(1,2)
    n2=randint(1,2)
```

24. At the end of your code, add the line:

```
plt.line(0, 0.5, games, 0.5, "arrow")
```

This line will draw a horizontal line across $y=0.5$. It might make it easier to copy your graph.

```
*evenorodds.py 40/40
total.append(t)
sleep(0.5)
print(ev,"",games,"=",prob[-1])
plt.window(0,games,0,1)
plt.text_at(1,"% even","left")
plt.text_at(13,"total games", "center")
plt.scatter(total,prob,"o")
plt.grid(games/10,0.1,"solid")
plt.line(0,0.5,games,0.5,"arrow")
```

Next, change the games variable to equal 500 instead of 10.

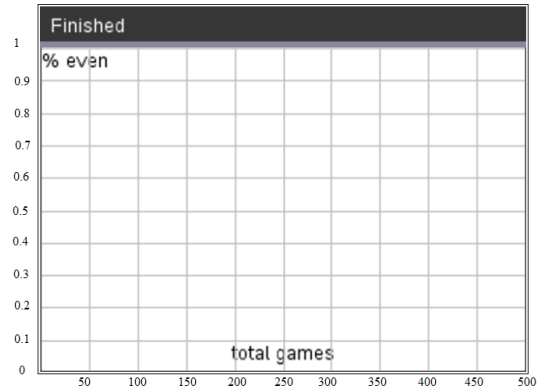
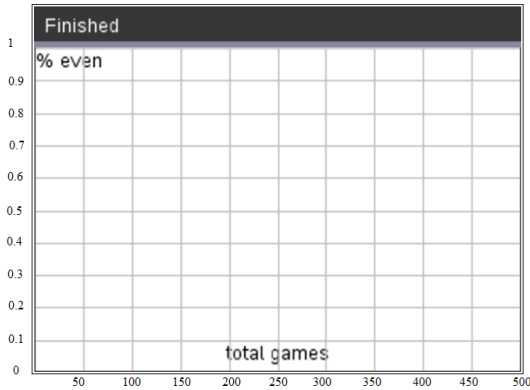
Simulating Fairness

TEACHER NOTES

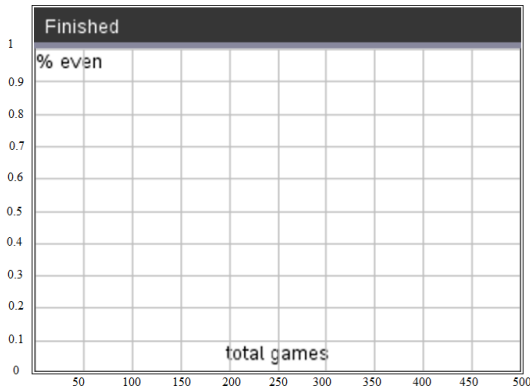
```
*evenorodds.py 22/39
if (n1+n2)%2==0:
    ev+=1
    #print(n1,n2,"even")
#else:
    #print(n1,n2,"odd")
p1.append(n1)
p2.append(n2)
prob.append(ev/t)
total.append(t)
#sleep(0.5)
```



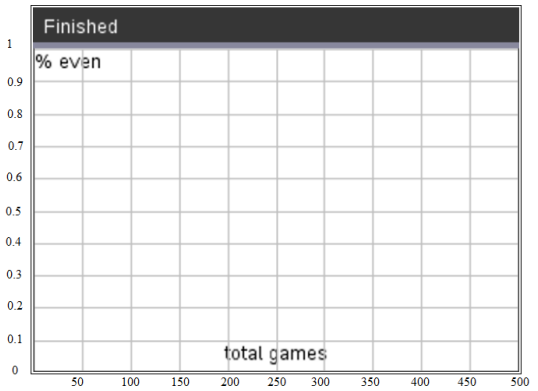
25. Run your program 4 times. Each time you execute your program, copy the graph onto a new graph below. Record the overall percent of evens as well.



_____ ≈
500



_____ ≈
500



_____ ≈
500

_____ ≈
500

Do you notice any patterns to your data?

What do you think would happen if you played 5000 games instead of 500?

What would the graphs look like?

What proportion of games would evens win?

The graphs above illustrate **The Law of Large Numbers**. The Law of Large Numbers states that the more games you play, the long-term probability will approach the theoretical probability. In this case, the theoretical probability “evens” will win the game is 0.5. Therefore, it doesn’t matter if you pick “evens” or “odds”, the probability of winning a game is 0.5 regardless your pick.

26. What if we change the rules of the game? What if you can play either a one, two, or a three? Would the game still be fair? Make a prediction. Do you think this new version would be fair? Why?

27. Modify your game. The players now can play a one, two, or a three.

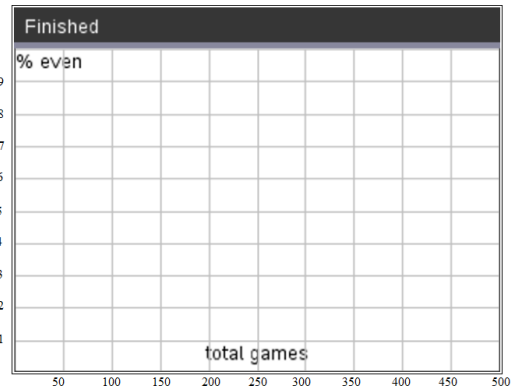
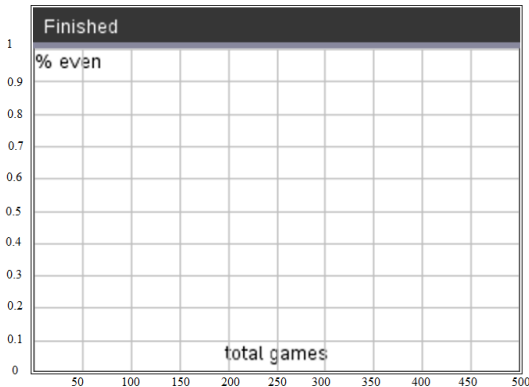
```
n1 = randint(1,3)
n2 = randint(1,3)
```



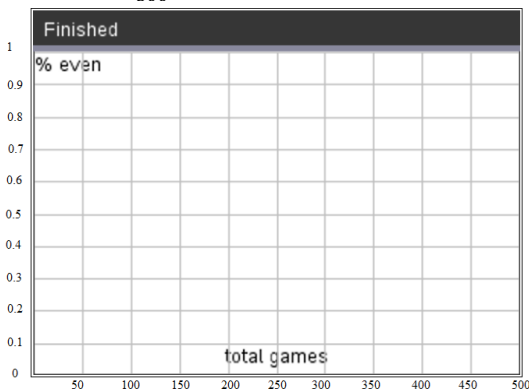
```
*evenorodds.py 20/40
ev=0
games=500

for t in range(1,games+1):
    n1=randint(1,3)
    n2=randint(1,3)
    print(n1,n2)
    if (n1+n2)%2==0:
        ev+=1
    #print(n1,n2,"even")
    #else:
```

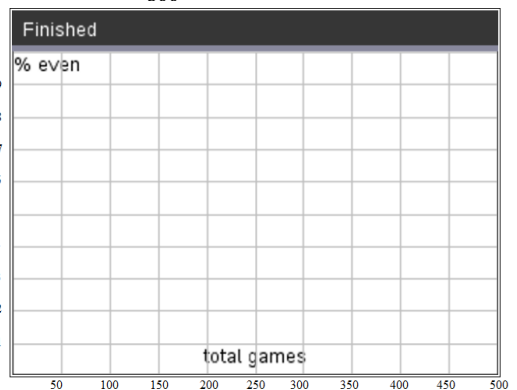
28. Run your **NEW** program 4 times. Each time you execute your program, copy the graph onto a new graph below. Record the overall percent of evens as well.



_____ ≈
500



_____ ≈
500



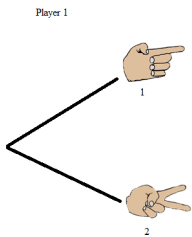
_____ ≈
500

_____ ≈
500

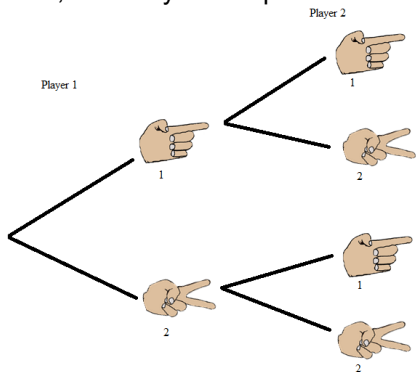
This version of the game is NOT fair. Why not? What does the theoretical probability of “evens” winning the new version?

29. How could we determine the probability without a simulation? One way would be to create a **tree diagram** that represents all the possible ways to play the game.

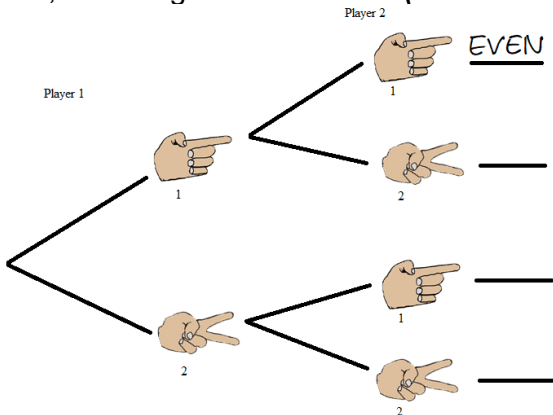
Step 1, Draw the possibilities for Player 1



Now, add Player 2's options.



Last, add the game outcomes. *(The first one is done for you. Add the other three outcomes.)*





34. Use the tree diagram above to answer the following questions:

How many possible outcomes are there? _____

$p(\text{sum even}) =$

$p(\text{sum odd}) =$

$p(\text{sum is } 3) =$

$p(\text{sum} < 4) =$

Does the probability of evens match your graphs from step #26?

Teacher Tip:

4 possible outcomes

$P(\text{sum even}) = 2/4$ $p(\text{sum odd}) = 2/4$ $p(\text{sum is } 3) = 2/4$ $p(\text{sum} < 4) = 4/4$

Answers will vary. The theoretical probability for 500 samples should be about 0.5 which matches the theoretical.

35. The graphs in step 29 showed the NEW game was not fair if the players were allowed to play a one, two or three.

Let's investigate why this version isn't fair.

a. Create a tree diagram for this version of the game. You don't have to draw the physical hands like the example above, you may choose to only put the numbers at the end of the line segments.

b. How many outcomes are possible in this version?

c. The game wasn't fair. Use the tree diagram to give the probability "evens" wins.

d. $p(\text{sum odd}) =$

$p(\text{sum is } 3) =$

$p(\text{sum} < 4) =$

$p(\text{sum} > 6) =$

Teacher Tip:

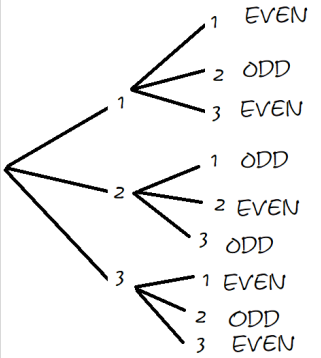


a.

b 9 outcomes

c 5/9 or 0.56

d. 4/9 or 0.44, 2/9 or 0.22, 3/9 or 0.33, 0



36. What would happen if:

player 1 could only play a 1 or a 2

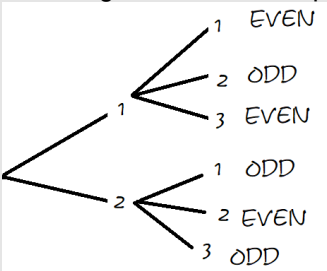
player 2 could play a 1, 2 or 3

a. Create a tree diagram for this version of the game.

b. Is this version of the game fair? Explain using your tree diagram.

Teacher Tip:

Yes the game is fair. The probability of “evens” winning is 3/6 or 0.5.



#Random Simulations

#=====

```

from math import*
from random import*
import tiplotlib as plt
from time import*

```

```
p1=[]
p2=[]

prob=[]
total=[]

ev=0
games=500

for t in range(1,games+1):
    n1=randint(1,3)
    n2=randint(1,3)
    print(n1,n2)
    if (n1+n2)%2==0:
        ev+=1
        #print(n1,n2,"even")
    #else:
        #print(n1,n2,"odd")

    p1.append(n1)
    p2.append(n2)
    prob.append(ev/t)
    total.append(t)
    #sleep(0.5)

print(ev,"/",games,"=",prob[-1])

plt.window(0,games,0,1)
plt.text_at(1,"% even","left")
plt.text_at(13,"total games", "center")
plt.scatter(total,prob,"o")
plt.grid(games/10,0.1,"solid")
plt.line(0,0.5,games,0.5,"arrow")
```