Pythagoras – Episode 1

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Teacher Notes & Answers

7 8 **9** 10 11 12

TI-Nspire | Investigation | Student | 30 min

Introduction

Prepare for an adventure filled with ancient knowledge and mathematical intrigue! Your journey begins with the enigmatic Pythagoras and the secrets hidden within his famous theorem. Legend has it that a group known as the Pythagorean Semicircle derived the formula but were uneasy with the implications as they unlocked deep mathematical truths about our number system.

Just like the treasure hunters Gates and Poole in the movie *National Treasure*, you will follow a trail of cryptic hints, geometric tools and mindful challenges. Each challenge will bring you one step closer to unravelling some of the incredible connections between Pythagoras (circa 570 BCE) and Fibonacci (circa 1170) and the incredible secret held by the Pythagorean Semicircle.

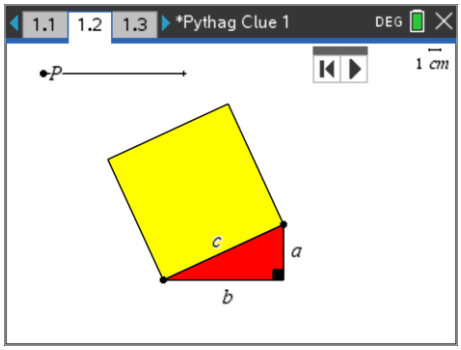
Some of the clues are well hidden, woven into a series of puzzles that require sharp thinking and teamwork.

Are you ready to begin this mathematical quest? Let's see where the first clue takes you!

Pythagoras

Open the TI-Nspire file: **Pythag Clue 1**

A mysterious animated key located on page 1.2 holds your first clue.



Clue 1:

A right-angled triangle with side lengths: a, b & c has been constructed. You can change the size and proportions of the triangle, but it will always remain a right-angled triangle. A square has been placed on the longest side of the triangle (hypotenuse).

Drag point P along the line to see how the key unfolds.

What is this mysterious animated key trying to tell us? Use the question prompts below to unravel the secret formula discovered by the Pythagoreans.

Question: 1.

In terms of the variables a, b and c:

a) Determine an expression for the area of the yellow square.
Answer: Area = c^2

b) Determine an expression for the area of the red triangle.
Answer: Area = ½ a b

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Question: 2.

With point P at the end of the line, the four triangles form a square (including the yellow region).

a) Explain how you know that the overall region formed is a square.
Answer: Each side of the overall square consists of the shorter side (a) plus the longer side (b), so each is the same length. The corners are the right angles formed by the right angles in the triangles.

b) Determine two expressions for the area of the square, one using its overall dimensions and the other as the sum of the individual areas.
Answer: Area = $(a+b)^2$ Area = 4 x ½ ab – c^2

c) Given the two expressions represent the same area, write an equation relating a, b and c only.
Answer: $(a+b)^2 = 4 \times ½ ab - c^2$ (By hand or CAS*): $a^2 + b^2 = c^2$.

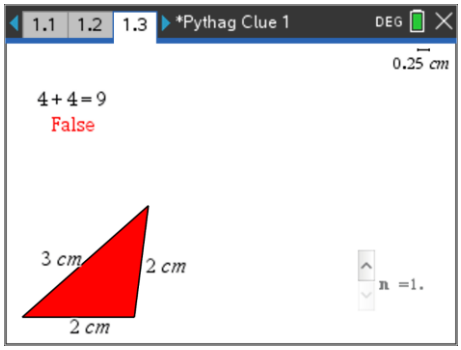
*CAS – Students will still need to provide prompts such as expand etc.

Navigate to page 1.3.

To make sense of the formula derived from the previous animated key and associated calculations, a series of triangles have been created. Substitute the side lengths of the triangle into your formula and see how they relate to the calculations on page 1.3.

Which one of the calculations produces a right-angled triangle?

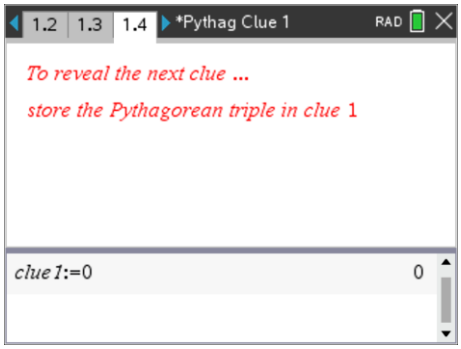
You will need this combination to proceed. Commit it to memory and then navigate to page 1.4.



There are two applications on page 1.4. The application in the top portion of the window contains a hidden clue. The application in the bottom portion of the window is a 'Calculator' application. You can use the mouse (track-pad) to activate the lower application or press Ctrl + Tab to toggle between the two applications.

The Pythagorean triple needs to be stored in 'clue1', currently the value of zero (0) has been stored. You can copy and past the current statement to a new line and insert the Pythagorean triple from the previous page.

Example: clue1:=456 would work if 4, 5, 6 was a Pythagorean triple.

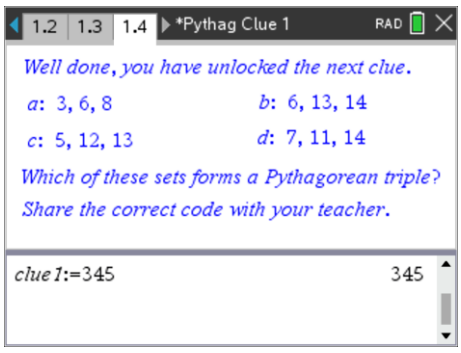


Once students have stored 'clue1' correctly they should see the following screen (opposite).

Students need to identify which of these 'triples' is a Pythagorean triple, with the answer being: 5, 12, 13.

The previous testing was done 'automatically', but in this instance the students need to do the calculators 'by-hand' or using a calculator application.

When students present their answer, it is a good opportunity to check they have at least a basic understanding of Pythagoras's theorem.



Once you have stored the correct value (triple) in clue 1, you will receive a brief challenge (top of screen). You need to identify the correct Pythagorean triple from four possibilities and take the result to your teacher.

If you have correctly identified the Pythagorean triple your teacher will supply you with the next TI-Nspire file and corresponding instructions.

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