Chaos

Introduction

Teacher Notes: There are a couple of ways to introduce this activity. Students can go outside and physically model the instructions. A tape measure, calculator, three objects (triangle vertices) and a class of students is all that is required. A PowerPoint slide set is also provided to help introduce the activity.

The activity title "Chaos" is chosen so as not to give away the surprising end result.

This activity involves repeatedly finding midpoints, starting with a point randomly placed inside a triangle. Once the first point has been placed, subsequent points are placed based on the following algorithm:

> A vertex is chosen at random. The new point is placed half way between the most recent point and the randomly selected vertex.

To help automate this process the triangle is placed on the Cartesian plane.

Question: 1.

Let the vertices A, B and C be located at: (0, 0); (6, 8) and (10, 2) respectively. The first point, P_1 is located at (4, 4)

a) Vertex A is selected at random. Determine the coordinates of the second point: P_2

Answer: Midpoint of A(0, 0) and P_1(4, 4) is P_2(2, 2)

b) Vertex B is now selected (random). Determine the coordinates of the third point: P_3

Answer: Midpoint of P_2(2, 2) and B(6, 8) is P_3(4, 5)

c) Vertex A is now selected (random). Determine the coordinates of the fourth point: P_4

Answer: Midpoint of P_3(4, 5) and A(0, 0) is P_4(2, 2.5)

d) Select a vertex at (random) and determine the coordinates of the fifth point: P_5

Answer: Answers will vary. Midpoint A and P_4 means P_5 = (1, 1.25); Midpoint B and P_4 means P_5 = (3.5, 4.625) or Midpoint of C and P4(6, 2.25)

e) Imagine this process is continued so that 200 points are plotted. What do you think the plot might look like?

Answer: Answers will vary.

Teacher Notes: The purpose of Question 1(e) is to establish whether students have any expectation of finding a pattern. In most cases students assume the points will most likely be randomly placed inside the triangle.

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Setting up the Triangle

Instructions:

Start a new document; insert a Graphs application.
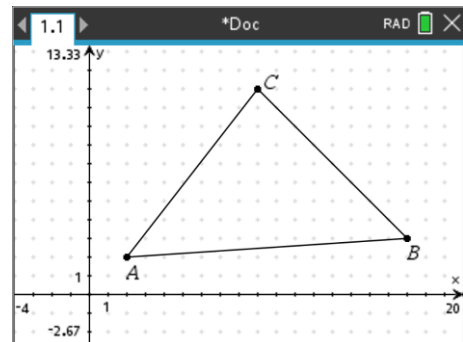
Use the zoom menu to focus on Quadrant 1.
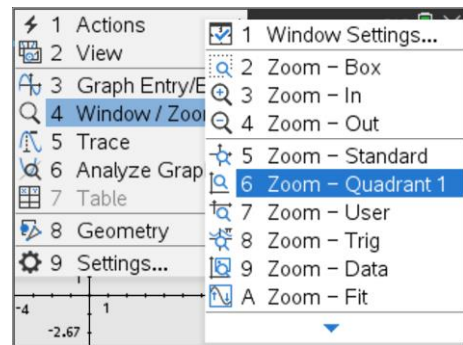
Use the Geometry menu to draw a triangle.

> **menu** > **Geometry** > **Shapes** > **Triangle**

If you want vertices A, B and C to have integer coordinates, switch on the grid and place each vertex on a grid point.

Vertex labelling can be switched on/off through the settings menu.

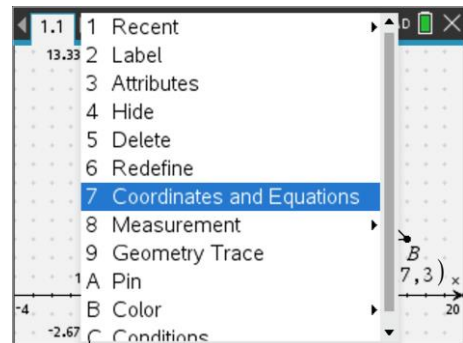The coordinates of each vertex are required for the program.

Move the mouse (cursor) over the top of a vertex then press:

> **ctrl** + **menu**

This will display the contextual menu shown opposite, select:
Coordinates and Equations.

Do this for each of the triangle's vertices.

The next task is to link each of the coordinates to variables that can be used in the program.
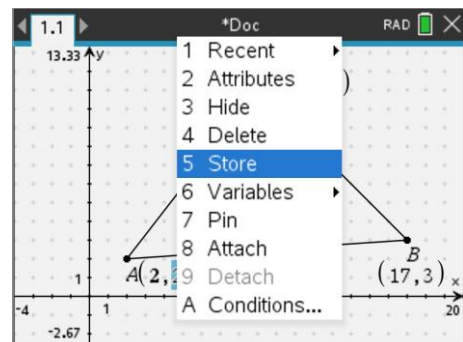
Move the mouse over the top of the x coordinate for point A, then press:

> **ctrl** + **menu**

This will display the contextual menu shown opposite, select:

Store

For the x coordinate (abscissa) of point A store the variable as XA.

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Repeat this process for each of the coordinates as follows:

    A (XA, YA)

    B (XB, YB)

    C (XC, YC)

Notice that each of the coordinates becomes **bold** once it is linked. Hovering the mouse over the coordinate confirms that it is linked to the corresponding variable.

An initial point needs to be plotted. It is not easy to confine the point to inside region of the triangle, however the individual user can ensure it is inside the triangle to begin and then explore what happens if the point is moved outside the triangle.

Place a point inside the triangle:

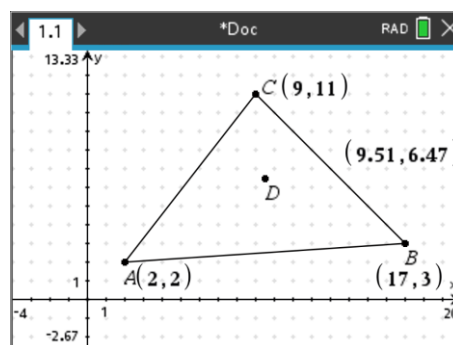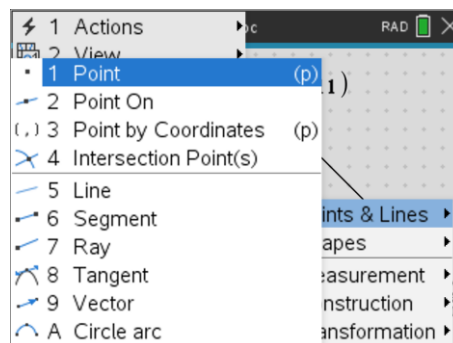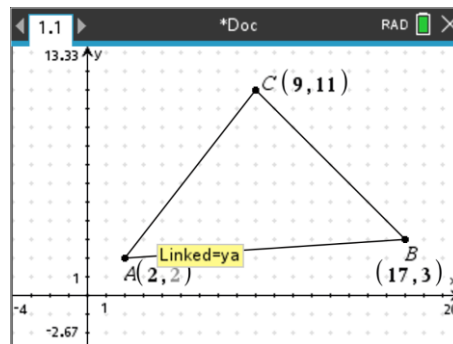   menu > **Geometry** > **Points and Lines** > **Point**

Note: A short-cut on TI-Nspire CX II is to simply press "P".

Display the coordinates of the point and store as follows:

    xv for x coordinate (abscissa)

    yv for y coordinate (ordinate)

By storing all the coordinates as variables, the program will be able to interact with the diagram.

Writing the Program

Insert a new page with the Program Editor. Call the program: Chaos.

The first task for the program is to identify how many midpoints will be plotted. A request statement could be used to input the number of points; however, it will be useful for automation purposes to have this value in the original definition of the program.

Navigate to the chaos() definition and insert an 'n'.

The number of points to be plotted is stored in 'n', two empty lists can be created to record these points. A list filled with zeros can be created with the sequence command: seq(expression, variable, start, low, high)

    xp:=seq(0,x,1,n)

Create another list for the y – points.

Note: The variable 'x' used to create the list is not used in this example but necessary for the syntax of the sequence command.

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
It is useful to keep track of the initial point (xv, yv). To avoid changing this in the program, the program will use (xm, ym) to represent all subsequent midpoints. While the first point is not a midpoint, it is useful to store xv in xm and yv in ym for the purpose of the loop (next step).

The number of times the loop needs to be executed is known (n), so a FOR loop is appropriate. Each time the loop is executed it will need to:

- Randomly select a vertex
- Calculate the new midpoint
- Store and update the current midpoint.

Insert a For loop:

> menu > **Control** > **For … EndFor**

The randint(lower,upper) command generates a random integer between the lower and upper limits. There are only three vertices to select from, so the random integer needs to vary between 1 and 3.

The randint() command can be typed directly or retrieved from the catalogue.

The randomly generated value needs to be stored so that it may be used to identify which vertex has been selected.

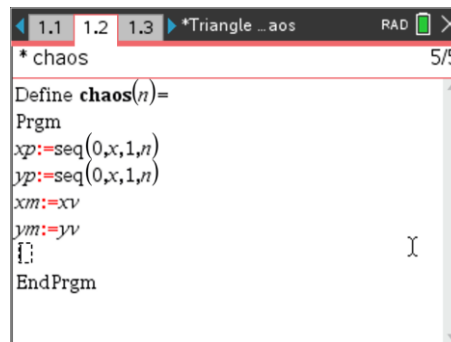If the random value is 1, then the new midpoint is halfway between the current value (xm, ym) and vertex A.

Examine the code included opposite to see how this is being done.

Add two more **If** … **Then** commands and include the instructions for vertex B or C.
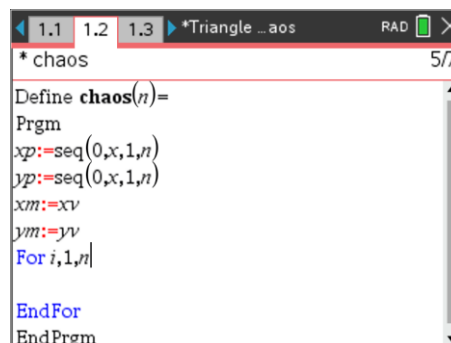

The program is almost finished. The coordinates need to be stored each time the loop is executed. The value of 'i' (loop counter) increments automatically each time the loop is executed. The lists xp and yp contain 'n' locations to store xv and yv.

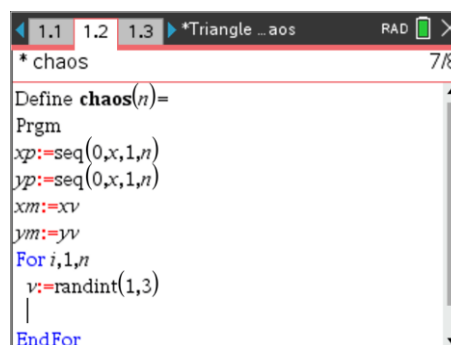Once these values have been stored the program is ready to run.

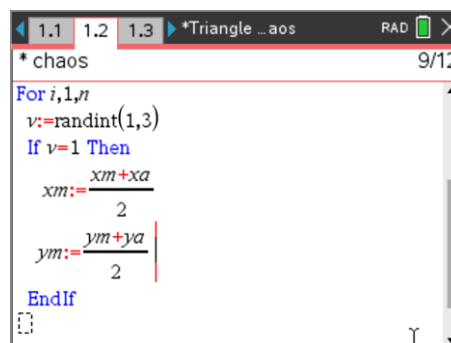Press **Ctrl** + **R** to run the program.
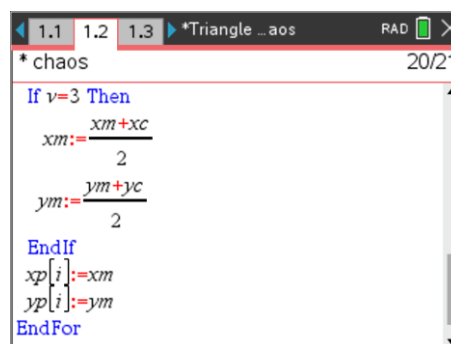
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Run the program and generate 50 points from a Calculator application.

chaos(50)

Navigate back to the Graphs application and create a scatterplot for the points (xp, yp).

Sample shown opposite.
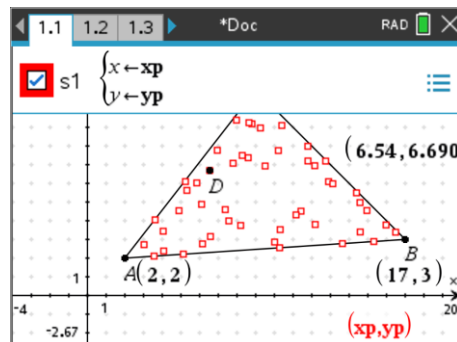
Question: 2.

An important stage in any programming task is to check the results.

a) Place the initial point close to the centre of the triangle and record the coordinates of this point.
Note: You can also select the coordinates and enter their values directly.

Answer: Answers will vary, but these coordinates are required for subsequent questions.

b) Run your program and generate 10 points. Navigate back to the Graphs application to see the scatterplot. Access the **Trace** > **Graph Trace** command from the menu, the coordinates of the first midpoint will be displayed. Write down the coordinates of the first midpoint.

**Answer**: Answers will vary. Sample shown below.
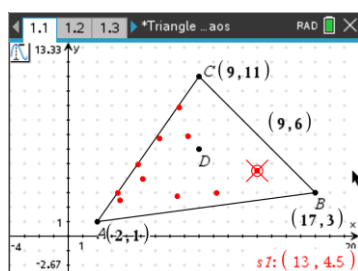
c) Write down the coordinates of your first point, the midpoint and identify which vertex was selected.

Answer: In the example shown first random point is B. $xm = \dfrac{9+17}{2} = 13$ & $ym = \dfrac{3+6}{2} = 4.5$

d) Using the trace tool, identify the second midpoint, write down the coordinates and confirm that they have been calculated correctly.

Answer: In the example shown first random point is B. $xm = \dfrac{13+2}{2} = 7.5$ & $ym = \dfrac{4.5+1}{2} = 2.75$

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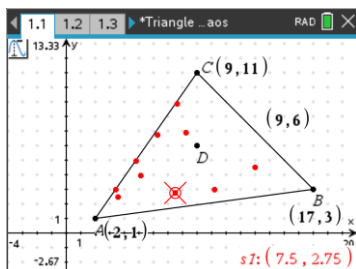
Question: 3.

Run your program 3 to 4 times generating 50 points each time. (The scatterplot will automatically update each time). Do you notice any pattern with the points or do they appear random / chaotic?

Answer: With just 50 points they appear to be completely random, it is highly unlikely that a student will see or identify any patterns with 50 points generated. [Three samples shown below.]
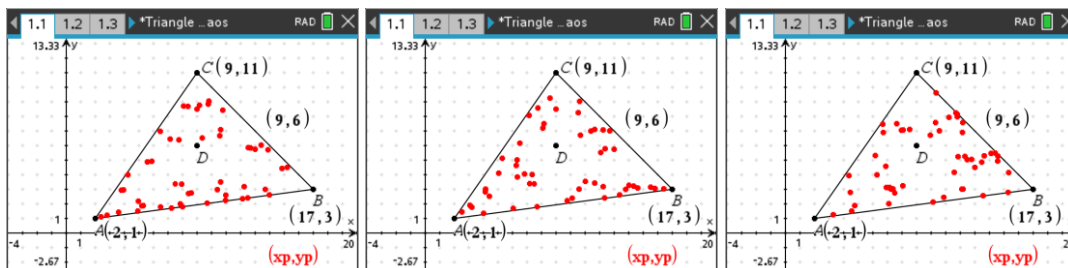

Teacher Notes: Calculator screen shots can be used to verify that students have completed this task.

Question: 4.

Run your program 3 to 4 times generating 150 points each time. (The scatterplot will automatically update each time). Do you notice any pattern with the points this time?

Answer: With 150 points patterns start to emerge. Students should notice that some regions inside the triangle do not seem to accumulate points. [Three samples shown below.]
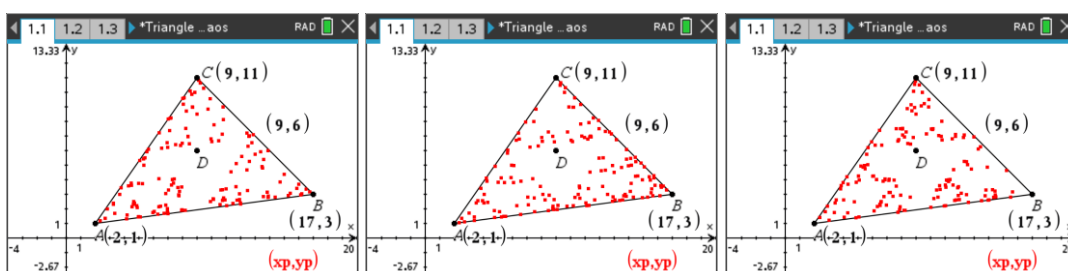
Teacher Notes: Students should notice the appearance of some regions that appear to be void, they may identify an 'upside-down triangle, depending on the points. Students may prefer to change the scatterplot to smaller points.

Students may also be encouraged to seed the problem by placing the initial point in this void area or even outside the triangle!

Question: 5.

Run your program 3 to 4 times generating 500 points each time. Do you notice any pattern with the points this time? Are they chaotic?

Answer: With this many points it should be clear that there is a central, inverted triangle that does not contain any points. Students should also notice that indeed there are several other triangular regions that do not accumulate points.

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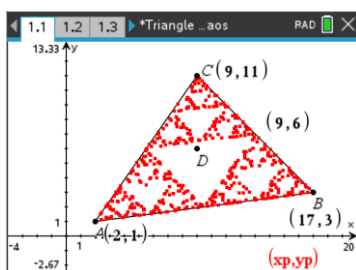
500 points   2000 points
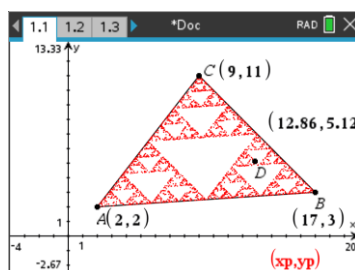

Small points used.

Teacher Notes: Students may identify the pattern as a Sierpinski triangle or Sierpinski gasket. Students may want to generate many more points, however the handheld platform will spend a considerable amount of time generating 500 points, 1000 points will take much to long on the handheld.

The 2000 point sample shown above was generated using the computer software.

Question: 6.

The vertices on the triangle are dynamic. Move points A, B and C to new locations then run the program again. Does the form of the triangle alter your answer to the previous question?

Answer: The form of the triangle does not affect the general result, a central, inverted triangle and lots of smaller triangular voids.
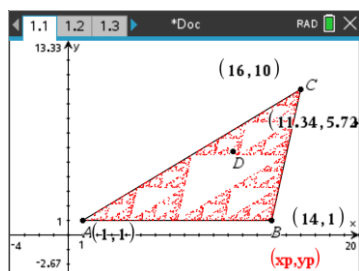
Teacher Notes: The program can be re-written to automatically execute by changing the input so that it is collected as chaos(n) rather than using the request command. Launch the program from a Maths-Box in the Notes application with approximately 250 points. Now the vertices of the triangle can be moved and the triangle will automatically repopulate. Similarly, students can move the original point and watch what happens.

For example, "What happens if the original point is outside the triangle?

Extension

The program can be launched from the Notes application within a Maths-Box. (Avoid using a large number of points!) Now what happens when you move the vertices or starting point from within the Graphs application?

What happens if instead of using a midpoint, the new points are weighted so that they are 1/3 of the distance to the vertex each time instead of the midpoint?
Hint: Consider using a weighted average.

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