

Ball Bounce



Teacher Notes & Answers

7 8 9 **10 11 12**



TI-Nspire™



Activity



Student



50 min

Equipment

- CBR(2)™ + Calculator to CBR USB cable
- Basketball
- Flat, level, hard surface on which to bounce the ball



Data collection only takes a minute or two so it is not necessary to have a one to one collection of CBRs. If you only have access to one CBR then students may be given some preparatory questions to review whilst waiting for their turn to collect data. Once students have collected their data they should save their document so that it is ready to use and their data is safe.

Each student or group will have their own data so equations will be unique; this ensures that every student is *doing their own work*. The easiest way to check student work is by either viewing it directly on their handheld, having them send you their document or by taking a series of screen captures.

There are many opportunities to extend this activity including modelling the sequence of ball bounces using a trigonometric function, using calculus to explore the velocity and acceleration of the ball or even determining the coefficient of restitution (Energy transfers). Check out the corresponding sections of the Texas Instruments Australia website for more activities or contact teacher-support@list.ti.com

With a well inflated basketball bouncing on a flat, hard and level surface the data is generally amazing! Allow students to experiment for some time getting their equation to fit *precisely* to the data. Each time they review their equation they are rehearsing the associated skills .

Introduction

The motion of a bouncing ball can be modelled using a parabola. In this activity you will collect position and time data using a CBR (Calculator Based Ranger) as a ball bounces beneath the detector. Things you need to keep in mind while completing this activity:

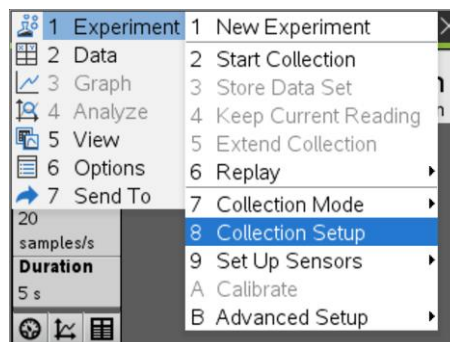
- The data is real, so it won't be 'perfect'. Real world data generally is messy! The idea of developing equations is to model the motion and evaluate the appropriateness and accuracy of the equations.
- It takes less than a minute to collect the data, so if you're not happy with your first data sample, you can quickly and easily generate a new sample.
- Keep the motion sensor at the same height throughout the data collection period. It can be moved side to side to track the bouncing ball, but the height cannot be changed. If the ball has a lot of side-ways movement, consider a more level surface or a smaller bounce height.
- Collect data for approximately 4 to 5 bounces. Images in this activity include sample data.

Set up

Start a new TI-Nspire document, insert a Calculator Application and then connect the CBR to the calculator using the USB cable. The Vernier DataQuest Application will launch automatically.

Once the Application has launched select:

Menu > Experiment > Collection Setup

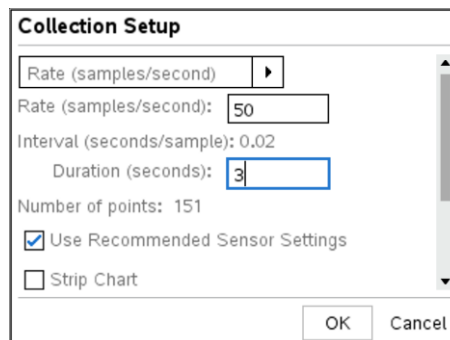


The data collection rate and duration depends on the type of ball and the height from which the ball is released. To begin try the following:

Rate: 50 (Samples per second)

Duration: 3 (Seconds)

Note that this menu also provides the option for remote data collection. If you are collecting the sample by yourself, check the box for remote collection so you can disconnect the CBR and use the Trigger button to initiate data collection.



A basketball bouncing on a smooth and level concrete surface makes an excellent target. If the ball is well inflated and released from a height of approximately 1m, the set up shown will produce between 3 to 5 bounces.

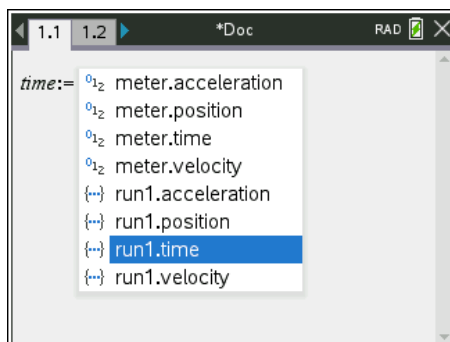
Navigate to the Calculator Application and assign the following variables:

time:= run1.time [Available from the VAR menu]

temp:= run1.position [Available from the VAR menu]

height:= max(temp) - temp

Note: If you collected and saved multiple data sets, select your best run set. run#.time



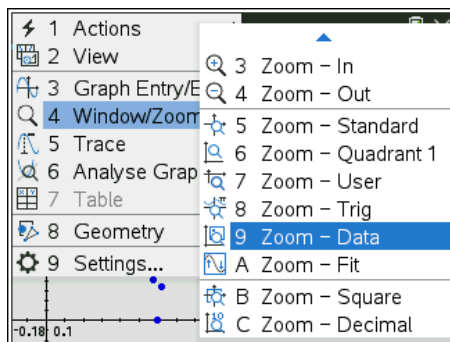
Navigate to the Graph Application and set up a **Scatter Plot** using the **Graph Entry/Edit** menu.

Set Scatter Plot 1 (S1) such that:

x = time

y = height

Use the Window / Zoom option to zoom in on the data. Make sure the x and y axis are visible, if not, adjust the window settings accordingly.



Question: 1

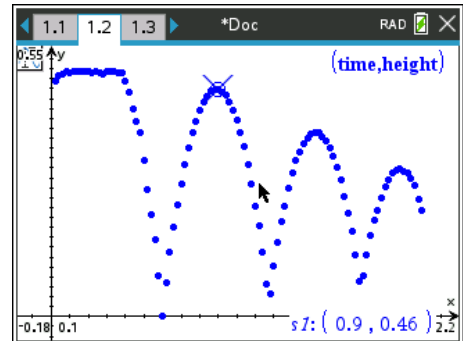
What is the purpose of the instruction: $\text{height} := \max(\text{temp}) - \text{temp}$?

As the motion is measured from the CBR, the ball bounces appear 'upside down'. The point of reference is the CBR, it is more natural for the ground to be the point of reference. The ground should be the furthest point (max) point measured from the CBR so the equation $\text{height} = \max(\text{temp}) - \text{temp}$ flips all the data to make the ground the reference point.

Modelling Motion using Translations

Use the **Trace > Graph Trace** command to move the cursor across the data set. Navigate to the top of the first ball bounce and record the time and height of the ball.

Note: Each data set is different so do NOT use the values shown opposite.

**Question: 2**

Use the translational form of a parabola: $f(x) = a(x-h)^2 + k$ to determine appropriate values for the corresponding parameters.

Answers will vary as each student has their own data.

The purpose of this question is for students to align the turning point focusing on h and k only.

Question: 3

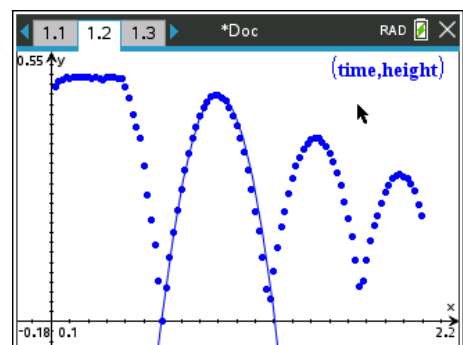
Graph your function and experiment with the value of a so that your parabola models the first ball bounce.

Note: You may choose to also experiment with h and k to create a better fit.

Answers will vary as each student has their own data. However, the a value for each student will be approximately -4.9 since $f(x)$ is measuring position so $f'(x) = 2a(x-h)$ is measuring velocity and $f''(x) = 2a$ is measuring acceleration. As the acceleration is due to gravity $2a = -9.8$

The graph shown opposite illustrates how well the parabola can model the bouncing ball. Further experimentation with the parameters in this equation could produce an even better fit.

Note: The data shown opposite includes a 'flat' spot at the start of the data collection. The flat spot corresponds to the ball being held after the CBR has started data collection.

**Question: 4**

Model the remaining ball bounces using the Trace command and the technique outlined in Questions 2 and 3.

Answers will vary as each student has their own data.

Question: 5

What similarities do each of the parabolic equations share?

The value of a should be approximately the same for each parabola (See Question 3)

Question: 6

Thinking about transformations and your current equation; suggest and test another way to generate equations for the remaining ball bounces.

If each equation is defined as $f(x)$ then subsequent equations can be defined as $af(x-h)+k$ or even $f(x-h)+k$ since the dilation for each is approximately -4.9 .



**Save your document! The data you have collected is unique, so too are your equations.
The accuracy of your equations is easy to check by using each of the graphs.**

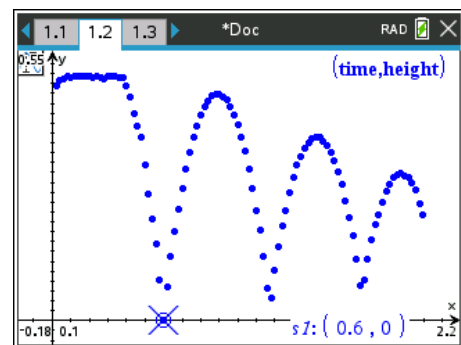
Modelling Motion using Impact Points

Another way to generate an appropriate equation is to consider *when* the ball hit the ground.

Switch the current equations OFF (don't delete them) so that you don't get confused between equations.

Use the **Trace > Graph Trace** commands to locate the closest point to *when* the ball struck the ground.

In the example shown here the first point is actually on the axis, the second point is not. The second point can be estimated using the proximity of neighbouring points.

**Question: 7**

Use the intercept form of a parabola: $f(x) = a(x-m)(x-n)$, determine appropriate values for the parameters m and n .

Answers will vary as each student has their own data.

Note that students will need to estimate the intercepts (times where the ball hit the ground). Students may also be asked to explore if a pattern exists between consecutive times. (Extension opportunity)

Question: 8

Graph your function and experiment with the value of a so that your parabola models the first ball bounce.

Note: You may choose to experiment with m and n to create a better fit.

While answers will vary, students should get an answer of approximately $a \approx -4.9$. Students should also be encouraged to reflect upon why this value is the same for this representation of the parabola.

$$a(x-m)(x-n) \approx a(x-h)^2 + k$$

$$ax^2 - mx - nx + mn \approx ax^2 - 2ahx + ah^2 + k$$

As the two quadratic equations are modelling the same data set the coefficient of x^2 should be approximately the same in each.

Question: 9

Explain why changing the value of a has no effect on the axis intercepts.

Null factor law: $a(x-m)(x-n) = 0$ has the same solutions regardless of the value of a .

Question: 10

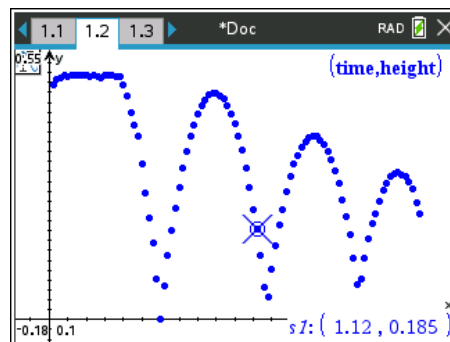
Model the remaining ball bounces using this technique and record your results. [Save your document]

Answers will vary as each student has their own data.

Modelling Motion using Simultaneous Equations

Another way to generate an appropriate equation is to identify appropriately spaced points along the path of the ball bounce and determining the equation that passes through these points.

Switch the current equations OFF (don't delete them) so that you don't get confused between equations. Use the **Trace > Graph Trace** commands to locate three points that you want the parabola to pass through. These points should be well spaced and representative of the path of the ball.

**Question: 11**

Set up the simultaneous equations using your selected three points. [Include the coordinates of your points]

Answers will vary as each student has their own data.

Note: If students are using the TI-Nspire CX CAS they may define $f(x) = ax^2 + bx + c$ and simply type: $y = f(x)$ using the corresponding coordinates to generate each equation. The equation can be stored as eqn1 and eqn2 ... so that they may be recalled easy.

Question: 12

Determine the equation that passes through your three points.

Answers will vary as each student has their own data.

Note: If students are using the TI-Nspire CX CAS they may use the solve command. Students using the TI-Nspire CX can use the solve system of equations option or use matrices and row reduction (RREF).

Question: 13

Determine the remaining equations for each of the ball bounces using this approach.

Answers will vary as each student has their own data.

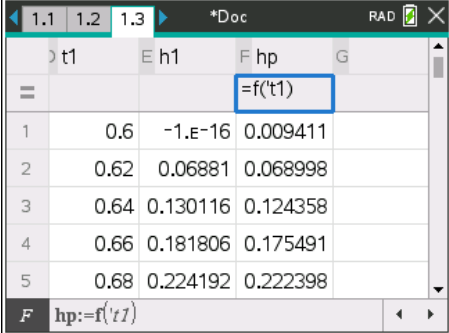
Extension

The three different approaches include various levels of difficulty, slightly different equations and presumably slightly better or worse models for the data. Which one is best? The purpose of this extension task is to determine which one of the equations provides the best model.

Suppose the equation for the first bounce is modelled by the function: $f(x)$. You can determine *where* the equation predicts the ball to be by considering the corresponding times. Locate the first and last point on the bounce using the trace command as before.

Insert a Spreadsheet Application and enter the list names at the top of each column. Suppose the bounce starts at time = 0.6 and concludes as time = 1.16. Select the cells containing this data, including the height values, (hold down the shift key) and copy (Ctrl + C) these values. Navigate to the top of an empty column (Ctrl + 7 = Top) and paste the data (Ctrl + V).

In a new column generate the corresponding values for the function as shown opposite. (t_1 = Times & h_1 = Heights for bounce one)



	t1	h1	hp
			=f('t1')
1	0.6	-1.E-16	0.009411
2	0.62	0.06881	0.068998
3	0.64	0.130116	0.124358
4	0.66	0.181806	0.175491
5	0.68	0.224192	0.222398

The spreadsheet shown opposite shows hp (height prediction) with the actual values (h_1) for the height of the first ball bounce, notice the accuracy of the predicted height throughout the ball bounce.

Use this approach to determine the 'average' difference between the predicted height and actual height for each ball bounce and its corresponding equations and discuss your findings.

Answers will vary as each student's data set is different. The idea here is that students need to create ways to avoid the summation of positive and negative differences that would produce an erroneous measure of the closeness of fit. Students may be encouraged to square the differences, sum them and then square-root the result.

Students should reflect and comment on the accuracy of the equation model for the ball bounces. As can be seen for the same data set show in the list above, the equation is generally accurate within +/- 5mm, depending on student data. This error is typically less than 5% and commonly averages 2% to 3%.

This section of the activity also provides students experience with editing data in the Spreadsheet application including working with functions and other lists commands.