

Watch Your P's and Q's

ID: 8975

Time required
30 minutes

Activity Overview

Students will use the Rational Zero Theorem to find all rational zeros of a polynomial.

Topic: Polynomials and Polynomial Equations

- *Divide one polynomial by another to obtain a quotient and remainder*
- *Prove and Apply the Remainder Theorem and Factor Theorem*
- *Approximate the real roots of a polynomial equation by graphing and identify the number of real roots*

Teacher Preparation and Notes

- *Students should have already begun to observe graphs of quadratic functions. Students should also have the ability to use synthetic division and the quadratic formula to solve polynomial equations.*
- *Remind students that the zeros of a polynomial are those values for which the polynomial is equal to zero ($f(x) = 0$).*
- *This activity is intended to be **teacher-led** with students in **small groups**. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "8975" in the keyword search box.***

Associated Materials

- *WatchPsAndQs_Student.doc*

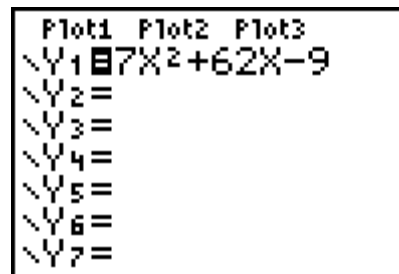
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *One of the Many Ways (TI-84 Plus family) — 11884*
- *Asymptotes and Zeros (TI-84 Plus family) — 9301*
- *Polly, Want Some Division? (TI-Inspire Technology) — 11607*

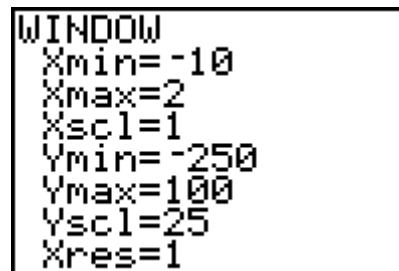
Problem 1 – Zeros of a Parabola

To find the zeros of the function $f(x) = 7x^2 + 62x - 9$ by graphing, first enter the function as **Y1**.



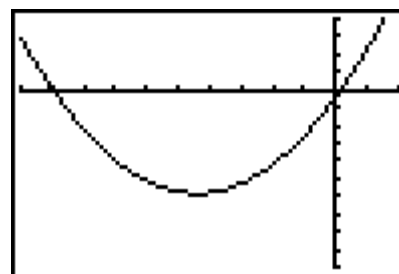
Adjust the graphing window to the settings shown.

You may wish to discuss with students why these settings are chosen and test out other settings.



Press **GRAPH** to view the graph.

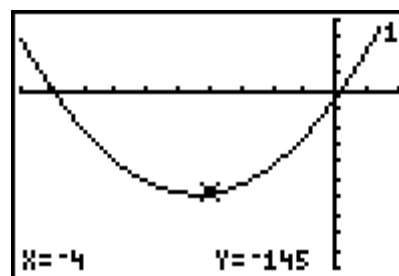
Note that the function is a quadratic function and has only two zeros. If the function had more zeros, we would not be able to see them in this window.



Press **TRACE**. Use the left and right arrows to move the cursor along the graph and locate the zeros.

The **Zeros** of a polynomial are where the **x-values** are zero. The location $(x, 0)$ is referred to as the *x-intercept*.

- **Students should see there are two zeros, at around -9 and 0 . Using the trace function, students will find the actual values are about -9 and between 0.08 and 0.21 . This is motivation for the use of the Rational Zero Theorem to find exact answers.**



Notice that these zeros are not exact. This is a limitation of finding results graphically. To find the exact value of this zero (if it is rational), the Rational Zero Theorem must be applied.

The **Rational Zero Theorem** states that all potential rational zeros of a polynomial are of the form $\frac{P}{Q}$, where P represents all positive and negative factors of the *last* term of the polynomial and Q represents all positive and negative factors of the *first* term of the polynomial.

For this polynomial, $7x^2 + 62x - 9$, the possible rational zeros are:

$$\frac{P}{Q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 7} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{9}{7}$$

We can find the exact zeros of the function by making a list. Press **STAT** then **ENTER** and enter all of the potential zeros into **L1**. You should have 12 entries.

L1	L2	L3	1
1	-----	-----	
-1			
3			
-3			
9			
-9			
.14286			
L1(1)=1			

Highlight **L2**. Enter **Y1(L1)** and then press **ENTER** to calculate the value of the function at each of these potential zeros.

L1	L2	L3	2
1	-----	-----	
-1			
3			
-3			
9			
-9			
.14286			
L2=Y1(L1)			

Scroll up and down the list. Where is the value of the function zero?

- What are the exact zeros of this function?

The table shows the exact zeros are -9 and $\frac{1}{7}$ (or 0.14286).

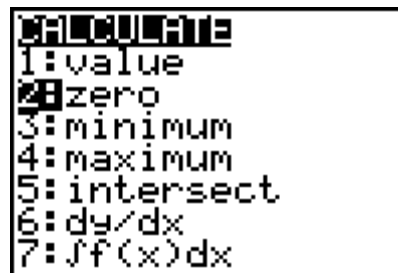
L1	L2	L3	2
1	60	-----	
-1	-64		
3	240		
-3	-132		
9	1116		
-9	0		
.14286	0		
L2(1)=60			

If using Mathprint OS:

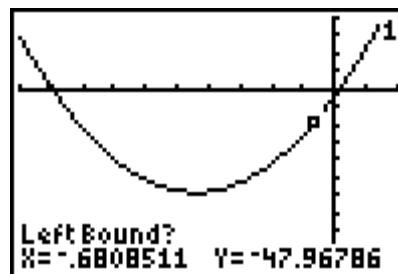
Students can display fractions in the list editor. For example, instead of $\frac{1}{7}$ being converted to the decimal .14286, it will appear 1/7. To do this, enter the value of the numerator, press **ALPHA** **[F1]** and select **n/d**. Then enter the value of the denominator and press **ENTER**.

L1	L2	L3	1
1	-----	-----	
-1			
3			
-3			
9			
-9			
1/7			
L1(7)=1/7			

You can also calculate the zeros of a graph using the **Zero** command. Press **2nd** **TRACE** to open the **Calculate** menu and chose **2:zero**.



Move the cursor to the left of the zero and press **ENTER**. Then move the cursor to the right of the zero and press **ENTER** again. Then make a guess and press **ENTER** again. The calculator displays the coordinates of the x-intercept.

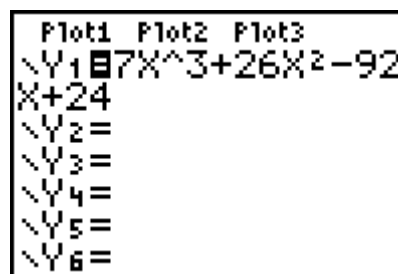


- Use the **zero** command to check your answers.

Problem 2 – Zeros of a cubic function

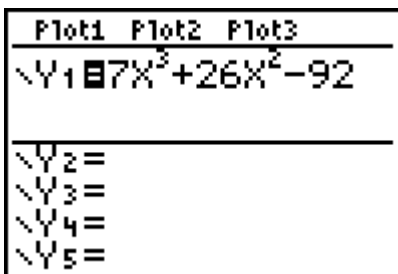
In this problem, you will find the zeros of a cubic function.

Enter the function $f(x) = 7x^3 + 26x^2 - 92x + 24$ in **Y1**.

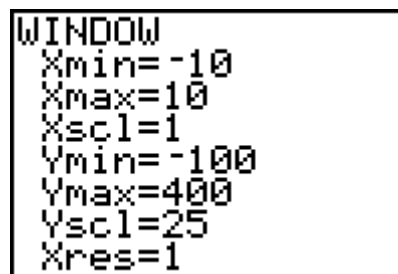


If using Mathprint OS:

When entering the function in Y1 and students press **^**, the cursor will move to the exponent position. Students should enter the value of the exponent and then press **▸** to move out of the exponent position.



Adjust the graphing window to the settings shown.



Trace the graph and locate the zeros.

- **The function has three zeros. From the graph, the points are at about $-6, 0.3,$ and $2.$**
- Identify all the possible rational zeros using the Rational Zero Theorem

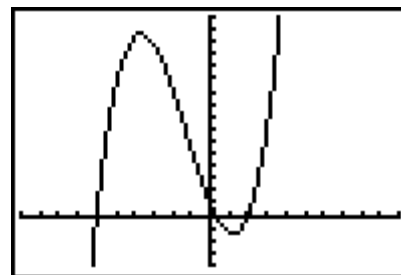
$$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1, \pm 7}$$

$$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

$$\pm \frac{1}{7}, \pm \frac{2}{7}, \pm \frac{3}{7}, \pm \frac{4}{7}, \pm \frac{6}{7}, \pm \frac{8}{7}, \pm \frac{12}{7}, \pm \frac{24}{7}$$

Enter these results in L1. (There should be 32 entries.)

- **The zeros are at $-6, \frac{2}{7}$ or $0.2857,$ and $2.$**



L1	L2	L3	Z
1	-35		
-1	135		
2	0		
-2	256		
3	171		
-3	345		
4	520		
L2(1) = -35			

Solutions

Problem 1 – Zeros of a parabola

- 2
- zeros around -9 and 0.15
- zeros at $x = -9$ and $\frac{1}{7}$

Problem 2 – Zeros of a cubic function

- 3
- zeros around $-3.2, 0.3,$ and 2.8
- $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{7}, \pm \frac{2}{7}, \pm \frac{3}{7}, \pm \frac{4}{7}, \pm \frac{6}{7}, \pm \frac{8}{7}, \pm \frac{12}{7}, \pm \frac{24}{7}$
- zeros at $x = -6, \frac{2}{7},$ and 2

Exercises

1. 2
2. Use synthetic division with the known zero of 2. This would reduce the polynomial to a quadratic which can be solved using the quadratic formula.
The remaining zeros are then $\frac{-5 \pm \sqrt{265}}{20}$.
3. Yes; yes. For example, the polynomial $f(x) = x^2 - 5$ has only irrational roots, and the polynomial $f(x) = x^2 + 5$ never intersects the x-axis and has no [real] roots.
4. 18.8 seconds