Transformations

Student Investigation

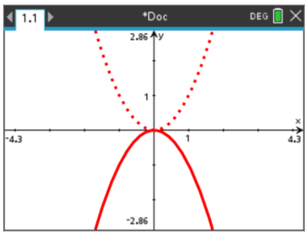
7 8 9 **10 11 12**

TI-Nspire™ Investigation Student 50 min

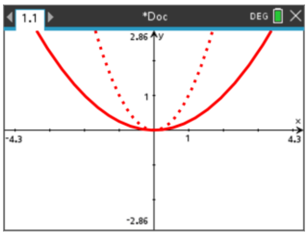
Introduction

Transformation means a change in form or appearance. Common transformations when dealing with functions include:

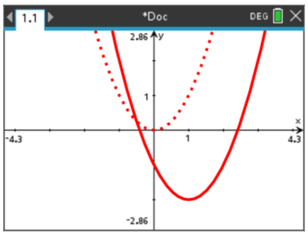
Reflection



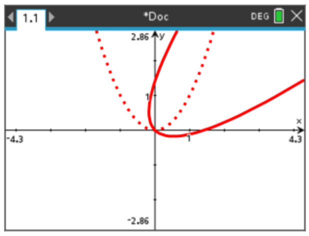
Dilation



Translation



Rotation

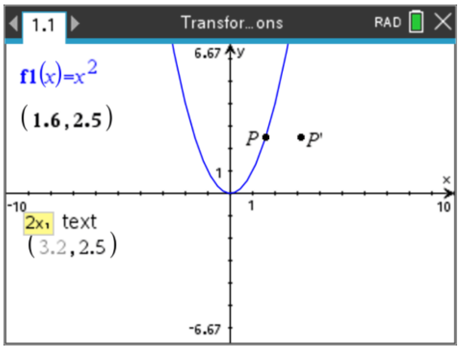


The aim of this activity is to provide an understanding of the algebra underpinning transformations. The technique involves the consideration of a single point and the effect it has on the general form or appearance of an entire family of points defined by a rule or function. A video tutorial is available to help set up your TI-Nspire file.

https://bit.ly/TI-transformations

Set up

Open your “Transformations” document created using the video link above.



Point P (x_1, y_1) is on the parabola: $f_1(x) = x^2$

Point P has undergone a transformation such that:

$P'(x', y')$ such that: $x' = 2x_1$ and $y' = y_1$

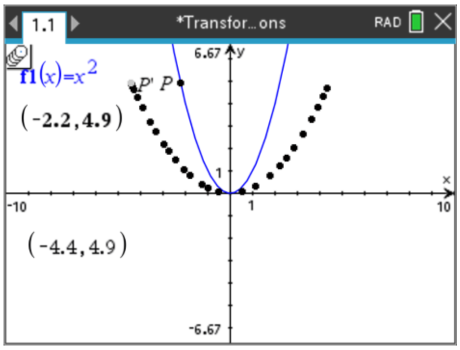
The text tip on P′ provides the transformation details.

Edit the transformation for your point P′ to match these conditions.

Drag point P along the parabola and observe the coordinates of P′.

Point P′ is described as a dilation, “parallel to the x axis” or “away from the y axis” by a factor of 2.

In the screen opposite, the path of point P′ has been traced using the Trace (Geometry) tool.



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Determining Equations

Question 1.

a) Given $x' = 2x$, $y' = y$ and $y = x^2$, determine the relationship between x' and y'. Check your answer using your calculator and the corresponding transformation tools on the calculator.

b) Based on your answer to the previous question, describe the transformation from $y = x^2$ to $y = 4x^2$. Test your answer using your calculator and the transformations file.

Question 2.

Edit the transformation for point P′ such that: $x' = x + 2$ and $y' = y$

a) Describe the location of point P′ in relation to P.

b) Determine the equation for the path of point P′.

Question 3.

Edit the transformation for point P′ such that: $x' = x$ and $y' = y - 3$

a) Describe the location of point P′ in relation to P.

b) Determine the equation for the path of point P′.

Question 4.

Point P is dilated by a factor of 3 away from the x axis, then translated 2 units in the negative x direction. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$.

Question 5.

Point P is translated by 2 units in the negative x direction, then dilated by a factor of 3 away from the x axis. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$.

Question 6.

Based on your answers to Questions 4 and 5, does the order of transformations matter?

Question 7.

$P(x, y)$ is transformed such that $x' = x$ and $y' = 2y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

b) Write an equivalent transformation, based on your equation in part (a).

Question 8.

$P(x, y)$ is transformed such that $x' = x - 3$ and $y' = -y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

b) State the corresponding transformations.

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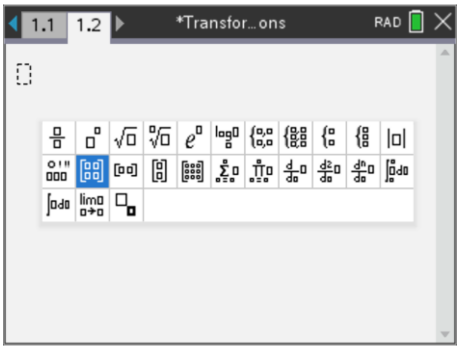
Extension

Transformations can also be described using matrices.

Insert a Calculator Application into your transformation document.

The matrices template can be found in the maths templates, notice the other matrices templates adjacent to this 2 x 2 template highlighted opposite.

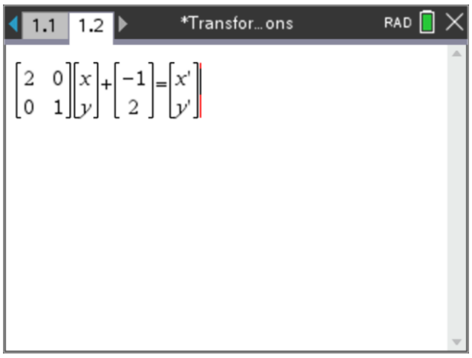


Enter the following equation using the corresponding matrix templates.

Note that the "primed" symbol: ′ can be obtained from the [?!▸] flyout menu.

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} -1 \\ 2 \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$$

Press [enter] to see the calculator result.



Question 9.

Use the calculator result to determine the equation for the 'matrix' transformations when applied to point $P(x, y)$ where $y = x^2$.

Question 10.

Use the calculator result to determine the equation for the 'matrix' transformation:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$ when applied to point $P(x, y)$ where $y = x^2$.

Question 11.

Use the calculator result to determine the equation for the 'matrix' transformation:

$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$ when applied to point $P(x, y)$ where $y = x^2$.

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