Name ____________________

Class ____________________

Logarithms are just another way of writing exponents. Just like exponents, logarithms have properties that allow you to simplify expressions and solve equations. In this activity, you will discover some of these properties with graphing and confirm them with algebra.

Problem 1 – The Power Property of Logarithms

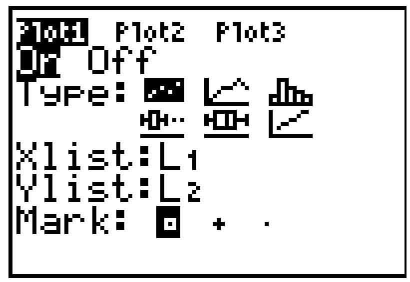
Suppose you wanted to simplify the logarithm of a power, like log a^2. How might you do it?

Let's start by defining a new variable $b = a^2$.You can make a table of values for *a* and *b* using the calculator's **List** feature.

Step a: Open the lists by pressing [STAT] [ENTER]. In **L1**, enter at least 10 values for *a*, that are in the domain of the logarithm function.

Step b: At the top of **L2**, enter a formula that will calculate $b = a^2$ from the values of *a* (**L1**).

Step c: Make a scatter plot of these values. Press [2nd] [STAT PLOT] and select **Plot1**. Adjust the settings to display the *a*-values in **L1** along the *x*-axis and the *b*-values in **L2** along the *y*-axis.

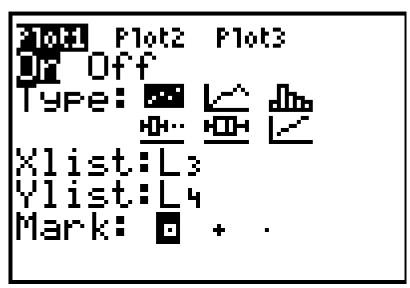


Step d: Press [ZOOM] and select **ZoomStat** to view the plot in an appropriate window.

1. What is the shape of the graph? Is this what you expected? Explain your answer.

Step e: Now we will define two new variables, *x* and *y*. Let ***x* = log *a*** and ***y* = log *b***.
At the top of **L3** enter a formula that calculates *x* from the values of *a* in L1.
At the top of **L4** enter a formula that calculates *y* from the values of *b* in L2.

Step f: Make a scatter plot of *y* vs. *x*. Press [ZOOM] and select **ZoomStat**.



2. What is the shape of the graph? Is this what you expected? Explain your answer.

Step g: The graph appears to be linear. Find the equation of a line through these points with the **LinReg(ax+b)** command (Press [STAT] and move to the CALC menu). After the command, type the *x* list, then a comma, then the *y* list.

3. What is the equation of the line through these points?

You should find that $y = 2x$. What does this mean? We can substitute to rewrite this as an equation in terms of a. The explanation for each step is given to the right.

$y = 2x$	(equation of the line)
$\log b = 2 \log a$	($x = \log a$ and $y = \log b$)
$\log a^2 = 2 \log a$	($b = a^2$)

Power Property of Logarithms

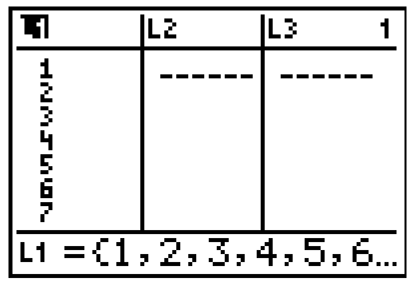
For $a > 0$, $\log a^b = b \log a$.

Examples

$\log x^3$ can be written as $3 \log x$

$8 \log x$ can be written as $\log x^8$

Problem 2 – The Product Property of Logarithms



Now suppose you wanted to simplify the logarithm of a product, like log 6*a*. How might you do it?

Start by defining a new variable ***b* = 6*a***.

Step a: Clear all the data from **L2**, **L3**, and **L4**, leaving the values for *a* in L1.

Step b: At the top of **L2**, enter a formula that will calculate $b = 6$ from the values of *a* (L1).

Step c: Use **Plot1** to make a scatter plot of *b* vs. *a*. Press $\boxed{\text{ZOOM}}$ and select **ZoomStat**.

4. What is the shape of the graph? Is this what you expected? Explain your answer.

5. Imagine a line through this data. What would be the slope and *y*-intercept?

6. Use the **LinReg(ax+b)** command to find the equation of a line through this data. Are the slope and *y*-intercept what you expected? Explain your answer.

Step d: Now define ***x* = log *a*** and ***y* = log *b***.
At the top of **L3** enter a formula that calculates *x* from the values of *a* in L1.
At the top of **L4** enter a formula that calculates *y* from the values of *b* in L2.

Step e: Make a scatter plot of *y* vs. *x*. Press $\boxed{\text{ZOOM}}$ and select **ZoomStat**.

The data appear linear. Find the equation of a line through these points with the **LinReg(ax+b)** command

7. What is the equation of the line through these points?

8. What is the *y*-intercept of the line?

You should have found that the equation of the line was $y = x + 0.778151$. Where did this y-intercept come from? (Here's a hint: Try raising 10 to the 0.778151 power.)

9. What is 0.778151?

10. Since $10^{0.778151} \approx$ ______, $\log(6) \approx$ ___________.

You have found that $y = \log 6 + x$. What does this mean? Substitute to rewrite this as an equation in terms of a. The explanation for each step is given to the right.

$y = \log 6 + x$	(equation of the line)
	($x = \log a$ and $y = \log b$)
	($b = 6a$)

Product Property of Logarithms

For $a > 0$ and $b > 0$, $\log ab = \log a + \log b$.

Examples

$\log xy$ is written in *expanded form* as $\log x + \log y$

$\log 7 + \log z$ is written as a single logarithm as $\log 7z$

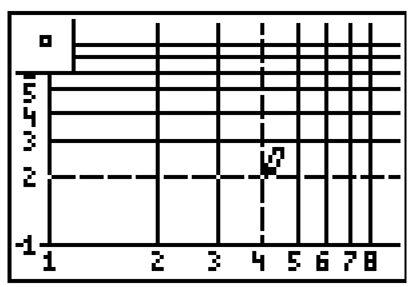
Problem 3 – The Quotient Property of Logarithms

In Problems 1 and 2, you used substitution to identify two properties of logarithms. A special kind of graph paper called log-log paper lets you make these substitutions graphically, by hand. In this problem, you will use log-log "paper" to simplify the expression $\log \frac{8}{a}$.

To see an example of log-log paper, launch the **Cabri^{TM}Jr** app and open the file **LOGLOG**.

11. Examine the grid. Why does the log scale start at 1 instead of 0?

Step a: Define $b = \frac{8}{a}$. Use the **Point** tool to plot points from the function $y = \frac{8}{x}$ on the log-log paper. Graph only points with whole number coordinates.



12. What is the shape of the graph?

Step b: Use the **Line** tool to draw a line through the points. The equation of this line with respect to the linear scale (not the log scale) is $y = 0.90309 - x$.

Where did this y-intercept come from? Can you guess? Press [2nd] [QUIT] to exit **Cabri^{TM}Jr** and make a calculation to confirm your guess.

13. What is 0.90309?

14. Since $10^{0.90309} \approx$ _____, $\log(8) \approx$ ________.

You have found that $y = \log 8 - x$. What does this mean? Substitute to rewrite this as an equation in terms of *a*. The explanation for each step is given to the right.

$y = \log 8 - x$	(equation of the line)
	($x = \log a$ and $y = \log b$)
	($b = \frac{8}{a}$)

Quotient Property of Logarithms

For $a > 0$ and $b > 0$, $\log \frac{b}{a} = \log b - \log a$.

Examples: $\log \frac{x}{y}$ is written in *expanded form* as $\log x - \log y$

$\log 7 - \log z$ is written as a single logarithm as $\log \frac{7}{z}$