



About the Lesson

This activity presents the normal probability distribution, beginning with the normal curve. Students explore the area under the curve between various x -values and use a model to find what percent of the area lies within 1, 2, and 3 standard deviations of the mean. They then apply this knowledge to answer questions about an orange crop with normally distributed weights. Percentiles are defined and calculated. As a result, students will:

- Determine the probability that a normally distributed variable will take a particular value in a given interval.
- Determine the x^{th} percentile for a normally distributed variable given its mean and standard deviation.

Vocabulary

- normal distribution
- standard deviation
- percentiles

Teacher Preparation and Notes


- This investigation is intended as an introduction to normal distribution for an Algebra 2 class. It could also be used in a precalculus or introductory statistics class.
- Students should have experience calculating the probabilities of simple and compound events. Experience calculating the area of compound shapes will also be helpful in completing the activity.

Activity Materials

- Compatible TI Technologies:

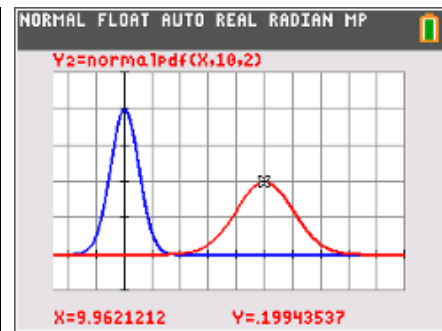
TI-84 Plus*

TI-84 Plus Silver Edition*

 TI-84 Plus C Silver Edition

 TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Average_Orange_Student.pdf
- Average_Orange_Student.doc



Part 1 – Exploring the Normal Curve

Students are to enter the function **normalpdf(x,0,1)** in **Y1**. (To type **normalpdf**, press **2nd** **vars** [distr] and choose it from the list.) In the **normalpdf** window, choose the **x value:** by pressing **[X,T,θ,n]** and leave μ and σ as their default settings of 0 and 1 respectively.

They will need to adjust the window settings.

Press **[graph]** to view the graph. Explain to students that this graph shows a normal curve, also known as a bell curve. The normal curve is one the most famous and useful functions in all of mathematics. It was developed by Abraham de Moivre in 1756 as an approximation for the binomial distribution.

The formula for the normal curve is $y = \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$.

However, there is no need for students to use this formula when making calculations; the calculator has built-in functions that will do the work.

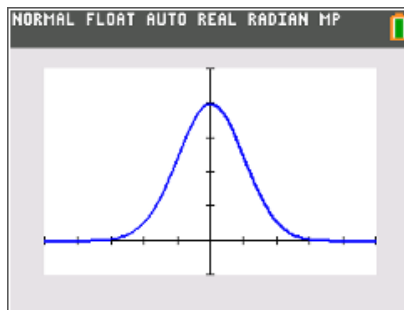
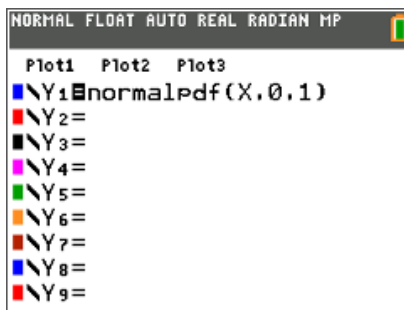
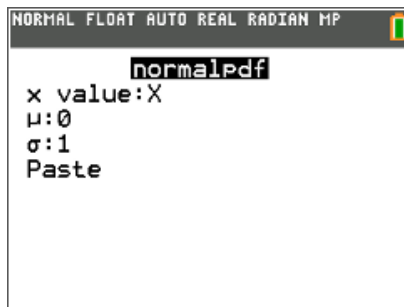
Students are to examine the shape of the curve.

1. Describe the normal curve in your own words.

Answer: Responses may vary. Students should see that it is symmetrical and extends to infinity in both directions. Its height is greatest in the center and decreases to the right and left, approaching (but never quite reaching) zero.

2. Why is the normal curve so useful?

Answer: The real-life distribution of many quantities can be approximated by this curve. Also, if many random samples are drawn from the same population, the averages will be normally distributed.

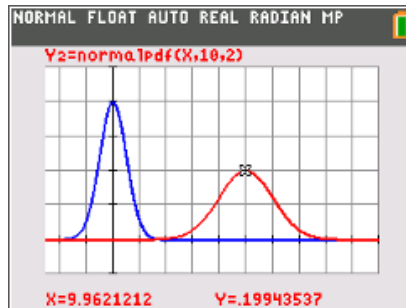
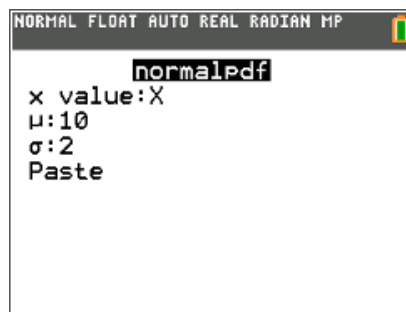




Students need to add the equation **normalPdf(X,10,2)** in **Y2**. Note the **normPdf(** command and its arguments: the random variable (x), the mean (10), and the standard deviation (2).

An appropriate window is $-5 \leq x \leq 20$ and $-0.1 \leq y \leq 0.5$ with an **xscl**: 2 and **yscl**: 0.1.

The shape of a normal curve is controlled by two parameters: the mean and the standard deviation.



Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing **2nd** **zoom** **[format]** to change the graph settings.

Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing **2nd** **ZOOM** to change the **[FORMAT]** settings. If your students are using TI-84 Plus, they could use GridDot.

- How is the shape of the normal curve affected by changing the mean? changing the standard deviation?

Answer: The mean controls the location of the hump of the curve, whereas the standard deviation controls whether the bell is broad and flat (larger standard deviation) or narrow and tall (smaller standard deviation).

If the bell curve represents the distribution of some measurement across a population, the mean is the population average and the standard deviation measures the spread of the population measurements.

Students are to explore the effect of these parameters on the curve by substituting different values into the **normalPdf(** command in **Y2**. To isolate the effect of each parameter, they should try keeping the mean constant and changing the standard deviation, and then keeping the standard deviation constant and changing the mean.

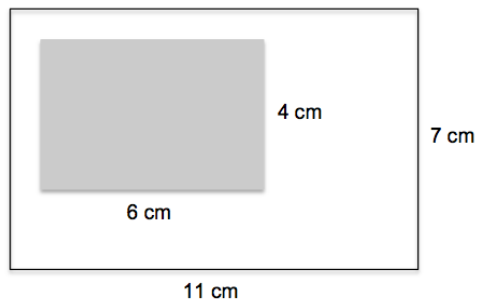
Be sure to explain to students that no matter what the mean and standard deviation, the total area under every normal curve is the same: 1.



Part 2 – Probability as Area

The idea of representing probability as area can be difficult for many students to grasp, especially when it is introduced simultaneously with the new concept of “area under a curve.” So we begin with a simpler example: the dartboard.

Ask students to imagine that they are randomly throwing darts at this board. The darts always hit the board, but where they hit is random; they are equally likely to hit any point on the board. How would you find the probability that a particular dart will land in the shaded area?



In this case, each outcome is represented by a point (where the dart hits) and we can find the number of outcomes by “counting the points,” also known as calculating an area.

What are the possible outcomes?

(All the points on the board, represented by an area of 77 cm².)

What are the desired outcomes?

(All the points in the shaded area, represented by an area of 24 cm².)

What is the ratio of desired outcomes to possible outcomes? ($\frac{24}{77} \approx 0.3117$)

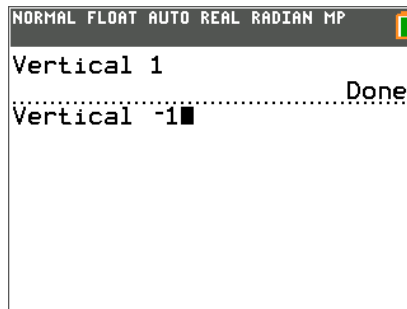
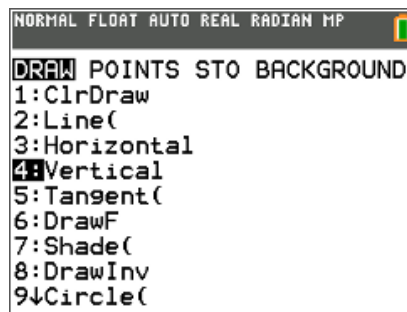
This is the probability that a dart will land in the shaded area.

Students need to delete the function from Y2 and then press **2nd** **mode** [quit] to return to the home screen.

From the Home Screen, students need to press **2nd** **prgm** [draw] to open the DRAW menu and choose **Vertical**.

Then students need to enter the command shown at the right to draw a vertical line at $x = -1$. Repeat these steps to draw a vertical line at $x = 1$.

After each command, students will be taken directly to the graph.

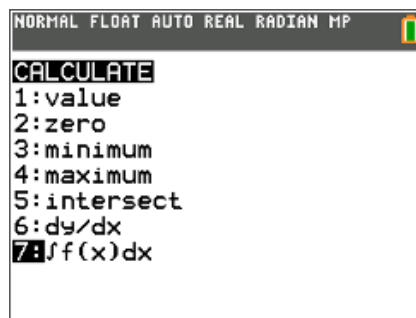




Students should examine the graph and see that the two vertical lines cut a “slice” from the center of the curve.

What is the area of the region below the normal curve, above the x-axis, and between these two lines?

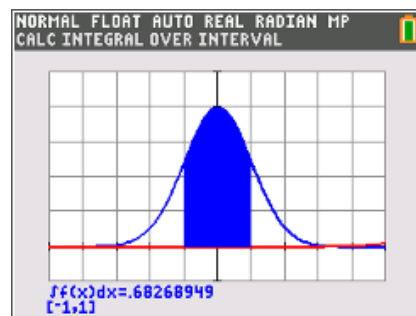
They can use the $\int f(x)dx$ command to find the area of the region below the normal curve, above the x-axis, and between these two lines. Press **2nd** **trace** **[calc]** to open the CALCULATE menu and choose it from the list.



This command is designed to find the area under a curve and above the x-axis. The equation of the curve is displayed at the top of the screen.

Type **-1** and press **enter** to set the lower limit. Then type **1** to set $x = 1$ as the upper limit. Students should resize their window according to their first normal curve, Y1.

The calculator shades and calculates the area. Because the standard deviation in this case is 1, the area shown is the area within one standard deviation of the mean, 0.



Recall that a distribution is a function that pairs probabilities and events. In the normal distribution, events are described in terms of x -values. The probability of a randomly selected x -value lying in the interval (a, b) is given by the area under the curve between a and b .

In other words, in the example above, the area that a randomly selected x -value is between -1 and 1 is about 68%.

4. What is the percentage of the area?

Answer: $\approx 68\%$

To help students understand why the probability is equal to this area, return to the “darts” analogy for probability. Imagine you are randomly throwing darts at a dartboard that is the shape of the normal curve. The probability that a dart lands on a point with an x -coordinate that is between a and b would be equal to the ratio of the shaded area shown and to the total area under the curve. Because the total area of every normal curve is 1, the probability is equal to the shaded area.



Students will use the **ClrDraw** command (2nd prgm [draw]) to clear the vertical lines and shading from the graph. Then they will use the $\int f(x)dx$ command to find the area under the curve that is with 1, 2, and 3 standard deviations of the mean.

Explain the result in terms of percentages. These results are a fast way to approximate probabilities from the normal distribution without calculating the area under the curve.

5. Generate a picture on your calculator to represent each of the following:
 - a. The area within 1 standard deviation of the mean
 - b. The area within 2 standard deviations of the mean
 - c. The area within 3 standard deviations of the mean

Answers: Part c is shown above and to the right.

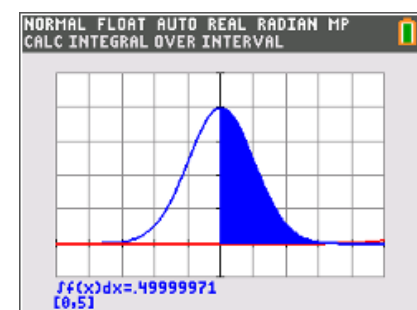
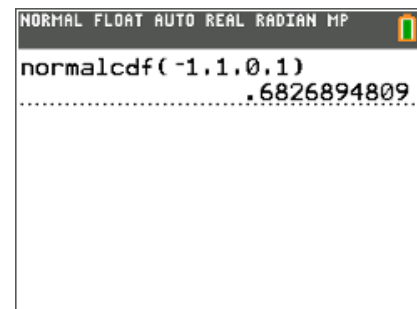
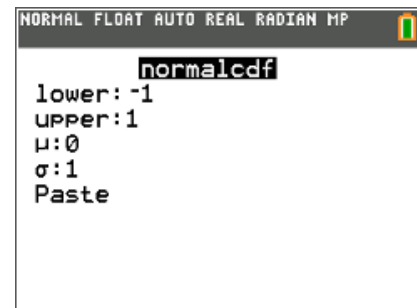
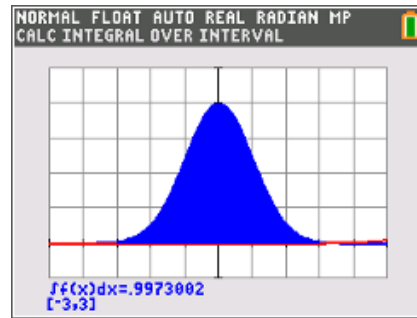
Another way to find these areas (which are also probabilities) is to use the **normalCdf**(command, found in the DISTR menu. Cdf stands for Cumulative Density Function.

This command finds the area under a normal curve between two x -values a and b for a given mean and standard deviation. The command in the screenshot finds the area between -1 and 1 under a normal curve with a mean of 0 and a standard deviation of 1 .

Students should use the **normalCdf**(command to verify the probabilities they found with the graph.

What is the area under the curve to the right of the mean?

Remind students that a normal curve extends to infinity. Both the $\int f(x)dx$ and the **normalCdf**(commands are numerical, so they cannot calculate the area of regions that are infinitely wide. Furthermore, with the $\int f(x)dx$ command, the upper and lower limits must be within the graphing window.





To find this area use the **normalCdf**(command and substitute 9E99 as an approximation of positive infinity. (You can also use $-9E99$ as an approximation of negative infinity.) To enter E, press $\boxed{2nd} \boxed{,} \boxed{[EE]}$. This will not find the exact area, but because the number is so large, and the height of the curve is so close to 0 this far away from the mean, it is very close.

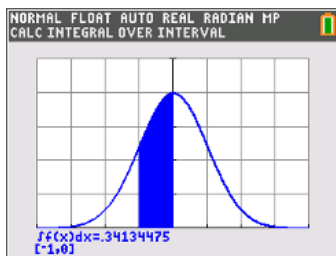
A simpler way to find this area is to divide the total area under the curve (1) in half!

Draw a picture to represent each of the following:

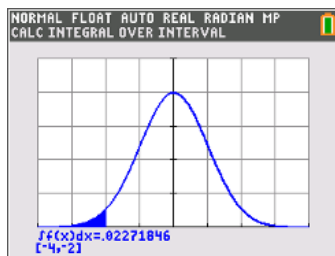
6. The entire area under the curve
7. The area to the left of the mean
8. The area to the right of the mean
9. The area from 1 standard deviation to the left of the mean to the mean
10. The area from the mean to 2 standard deviations to the right of the mean
11. The area to the right of a line 3 standard deviations to the right of the mean
12. The area to the left of a line 2 standard deviations to the left of the mean
13. The area to the right of a line 1 standard deviation to the left of the mean

Sample Answers:

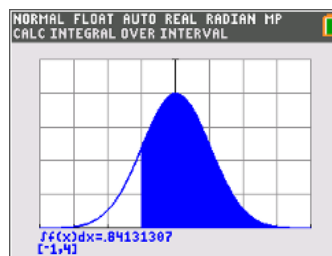
9.



12.



13.





Part 3 – Application of the Normal Distribution

Now we turn to a real-life variable that is normally distributed.

A farmer harvests a crop of oranges. The weights of the oranges are normally distributed with a mean of 310 grams and a standard deviation of 15 grams.

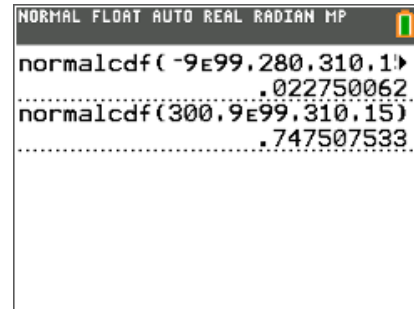
Students are to graph the distribution in an appropriate window.

To find the percent of the oranges that weigh 280 grams or less, students should realize that 280 is 2 standard deviations away from the mean (310–15–15). Since 95% of the area lies within 2 standard deviations of the mean, the total area under the curve is 1, and the curve is symmetrical, the percent of oranges weighing less than 280 grams is

$$P(x < 280) = \frac{1 - 0.9545}{2} = 0.02275 = 2.275\%.$$

Once students have used geometric logic to solve this problem, demonstrate another way. In this case, the command **normCdf(-9E99, 280, 310, 15)** yields the same result.

Students can use the **normCdf(** command or **∫f(x)dx** command to solve what percent of oranges will be sold to the commercial buyer.

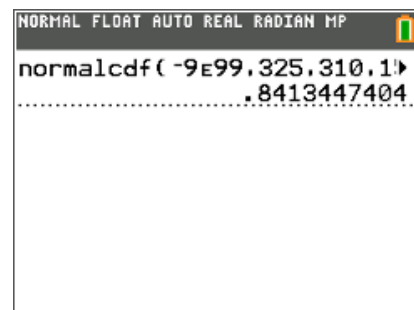
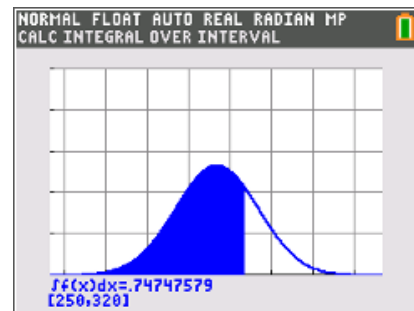


Explain to students that a value in the x^{th} percentile is greater than $x\%$ of the values in the set. For example, a test score in the 95th percentile is equal to or better than 95% of the scores.

Students should use either the **normCdf(** or **∫f(x)dx** commands for calculating the area under the curve to the left of 320 grams.

An appropriate window could be $250 \leq x \leq 370$ and $0 \leq y \leq 0.05$ with an **xscl:** 15 and **yscl:** 0.01.

To find the weight of an orange at the 84th percentile, student should recall that 68% of the area under a normal curve is within 1 standard deviation of the mean. Half of 68% is 34%, so 34% of the area is within one standard deviation and to the right of the mean. Half of the curve, or 50%, is to the left of the mean, so the area from negative infinity to one standard deviation above the mean is 84%. An orange weighing $310+15 = 325$ g is at the 84th percentile. Students should use the **normCdf(** or **∫f(x)dx** commands to check their answers.





14. Sketch the graph of the normal distribution for this data. Be sure to label the axes and scale.

Answer: The graph should resemble a bell-shaped curve in which the top of the curve occurs at $x = 310$. The x -axis should be segmented in intervals of 15 on either side of 310.

15. The farmer sells all the oranges weighing 280 grams or less to a juicer. What percent of the oranges will be sold to the juicer? (Hint: Use your results from Part 2.)

Answer: $\approx 2.5\%$

16. The farmer sells all the oranges weighing 300 grams or more to a commercial buyer. What percent of his oranges will be sold to the commercial buyer?

Answer: $\approx 74.75\%$

17. What does it mean if a test score is at the 75th percentile?

Answer: It means that particular test score is equal to or better than 75% of the scores.

18. What is the percentile rank of an orange weighing 320 grams?

Answer: $\approx 74.75\%$

19. What is the weight of an orange at the 84th percentile?

Answer: ≈ 325 grams