Scalar Resolute

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Answers

TI-Nspire   Investigation   Student   45 min

Introduction

The scalar projection of a vector onto another vector is also referred to as the scalar resolute. To understand the scalar resolute, begin by working and thinking in a two dimensional geometry environment, take this understanding and use it to work with vectors and vector equations in three dimensions.

For more examples, instructions and concept development, watch the video:
http://bit.ly/ScalarResolute

Scan to Watch

Scalar Resolute

Open the TI-*n*spire document: "Scalar Resolute". The first page contains some introductory information. Navigate to page 1.2 to explore how the magnitude of vector $\underset{\sim}{a}$ effects the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$.

To grab vector $\underset{\sim}{a}$ or $\underset{\sim}{b}$ move the mouse over the head of the vector and hold down the mouse button or press Ctrl then Click. To release the grip (hand) press ESC.

The vectors have been set up so that the angle between them can only be change by using the "Angle" slider.

The scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ is shown as a thick line.

To reveal information about the construction of the scalar resolute, click on the 'show' button. When this information is revealed it may be harder to grab the head of vector $\underset{\sim}{a}$. The screen text tip will display 'segment', press TAB to change to 'vector'.
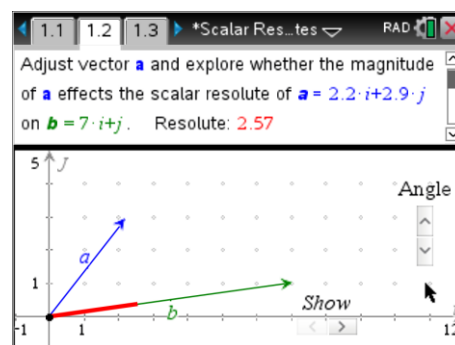
Question: 1.

Change the magnitude (length) of vector $\underset{\sim}{a}$ and observe changes to the scalar resolute. How does the magnitude of $\underset{\sim}{a}$ change the scalar resolute?

Example: Would doubling the magnitude of $\underset{\sim}{a}$ double the scalar resolute?

Increasing/decreasing the length or magnitude of vector $\underset{\sim}{a}$ causes a corresponding increase/decrease in the scalar resolute. Doubling the size of vector $\underset{\sim}{a}$ would cause the resolute to double in size also.

The angle between the two vectors can be changed using the "Angle" slider. The exact value of the angle is not the focus, rather how it affects the scalar resolute. Change the angle by increasing or decreasing the slider and observe the changes to the scalar resolute.

Question: 2.

Under what circumstances will the scalar resolute equal zero?

Author: P. Fox

TEXAS INSTRUMENTS

If the angle between the two vectors is 90° the scalar resolute will be zero, because no component of vector $\underset{\sim}{a}$ will be in the direction of $\underset{\sim}{b}$.

Question: 3.

Under what circumstances will the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ be equal to $\left|\underset{\sim}{a}\right|$?

If the angle between the two vectors is 0° the scalar resolute will be the same as $\left|\underset{\sim}{a}\right|$.

Question: 4.

If $\left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right|$ and the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ is equal to $\dfrac{1}{2}\left|\underset{\sim}{b}\right|$, determine the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

A diagram of this situation reveals that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ form two sides of an equilateral triangle; therefore the angle must be 60°. Using vector notation:

$$\left|\underset{\sim}{a}\right|\cos(\theta) = \frac{1}{2}\left|\underset{\sim}{b}\right| \qquad \textit{Given scalar resolute is equal to } \frac{1}{2}\left|\underset{\sim}{b}\right|$$

$$\cos(\theta) = \frac{\left|\underset{\sim}{b}\right|}{2\left|\underset{\sim}{a}\right|} \qquad \textit{Transposing equation...}$$

$$\cos(\theta) = \frac{\left|\underset{\sim}{b}\right|}{2\left|\underset{\sim}{b}\right|} \qquad \textit{Given information in question: } \left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right|$$

$$\cos(\theta) = \frac{1}{2}$$

Question: 5.

Let θ be the angle between vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, such that $0 \le \theta \le \dfrac{\pi}{6}$. If the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ is greater than $\left|\underset{\sim}{b}\right|$ determine the minimum value for $\left|\underset{\sim}{a}\right|$ in terms of $\left|\underset{\sim}{b}\right|$?

Students may use geometry to resolve this problem and see that $\left|\underset{\sim}{a}\right| > \dfrac{2\left|\underset{\sim}{b}\right|}{\sqrt{3}}$

Alternatively, from previous results, if the angle between the vectors is 0° then $\left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right|$, however as the angle increase the size of the scalar resolute decreases. When $\theta = \dfrac{\pi}{6}$ then $\left|\underset{\sim}{a}\right|\cos\left(\dfrac{\pi}{6}\right) > \left|\underset{\sim}{b}\right|$

therefore $\left|\underset{\sim}{a}\right| > \dfrac{2\left|\underset{\sim}{b}\right|}{\sqrt{3}}$ in order to ensure the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ is greater than $\left|\underset{\sim}{b}\right|$

Question: 6.

Use the 'hint' slider (show) to view a geometrical construction of the scalar resolute. Use this information to determine a rule for the scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$.

Hint: Think in terms of a simple two dimensional geometry environment and remember that the 'length' of $\underset{\sim}{a}$ is written as: $\left|\underset{\sim}{a}\right|$. Use the vectors and result on page 1.2 to check your rule.

Author: P. Fox

TEXAS
INSTRUMENTS

Diagram shows geometrical view so scalar resolute of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ equals: $\left|\underset{\sim}{a}\right|\cos(\theta)$

Note: Using $\underset{\sim}{a}\cdot\underset{\sim}{b}=\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos(\theta)$ means this can be written as: $\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\left|\underset{\sim}{b}\right|}$ so the scalar resolute can be

computed without knowing the angle between the two vectors.

Scalar Resolute - Calculations

The vectors on page 1.2 are stored in variables **a** and **b**.
Navigate to page 1.3, a calculator application.
Calculate the dot product of vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.
Menu > Matrix and Vector > Vector > Dot Product
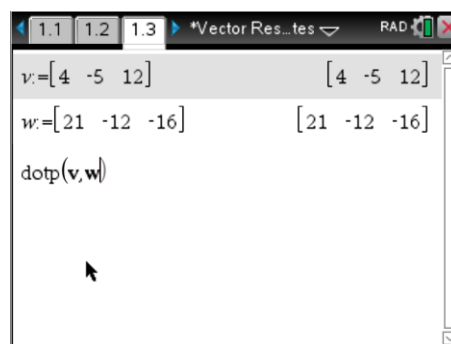A comma is used to separate the two vectors in the Dot Product command.
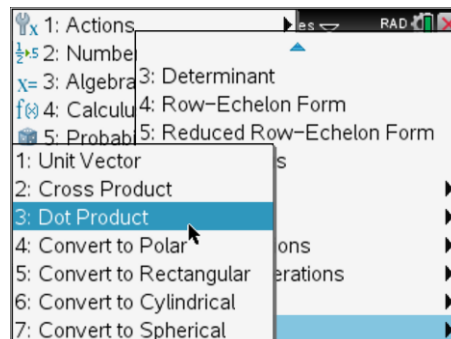
Define the following vectors using matrix notation:

$$\underset{\sim}{v}=4\underset{\sim}{i}-5\underset{\sim}{j}+12\underset{\sim}{k}$$

$$\underset{\sim}{w}=21\underset{\sim}{i}-12\underset{\sim}{j}-16\underset{\sim}{k}$$

Use dotp(v,w) to determine the dot product.

The magnitude of a vector can be calculated using:
$$\left|\underset{\sim}{v}\right|=\sqrt{\underset{\sim}{v}\cdot\underset{\sim}{v}}$$
The scalar resolute of vector $\underset{\sim}{v}$ on $\underset{\sim}{w}$ is therefore:

$$\frac{dotp(v,w)}{\sqrt{dotp(w,w)}}$$

Alternatively, use norm(v) to calculate $\left|\underset{\sim}{v}\right|$
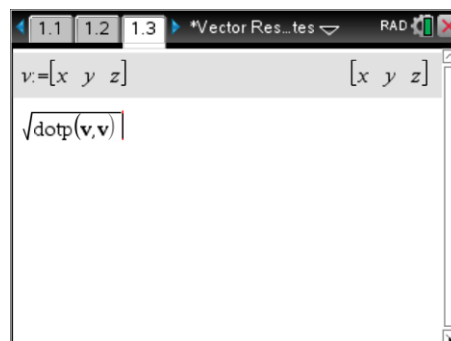
Question: 7.

Use the calculator to determine: dotp(v,unitV(w)) and explain why the result is the same as the scalar resolute.

Same as scalar resolute because $\dfrac{\underset{\sim}{v}\cdot\underset{\sim}{w}}{\left|\underset{\sim}{w}\right|}=\underset{\sim}{v}\cdot\left(\dfrac{\underset{\sim}{w}}{\left|\underset{\sim}{w}\right|}\right)$ where $\left(\dfrac{\underset{\sim}{w}}{\left|\underset{\sim}{w}\right|}\right)=\hat{\underset{\sim}{w}}$

Question: 8.

Let $\underset{\sim}{v}=4\underset{\sim}{i}-3\underset{\sim}{j}$ and $\underset{\sim}{w}=x\underset{\sim}{i}+y\underset{\sim}{j}$, if $\left|w\right|=12$ and the scalar resolute of v on w equals zero, determine the values of x and y.

Author: P. Fox

TEXAS INSTRUMENTS

$$|\underset{\sim}{w}| = \sqrt{x^2 + y^2}$$

$$12 = \sqrt{x^2 + y^2}$$

$$144 = x^2 + y^2$$

and

$$\frac{(4\underset{\sim}{i} - 3\underset{\sim}{j}) \cdot (x\underset{\sim}{i} + y\underset{\sim}{j})}{\sqrt{x^2 + y^2}} = 0$$

$$4x - 3y = 0$$

Simultaneous Equations

$$x = \pm\frac{48}{5} \quad and \quad y = \mp\frac{48}{5}$$

Note that students may also recognise if the scalar resolute is zero then the vectors are perpendicular.

Author: P. Fox

TEXAS INSTRUMENTS