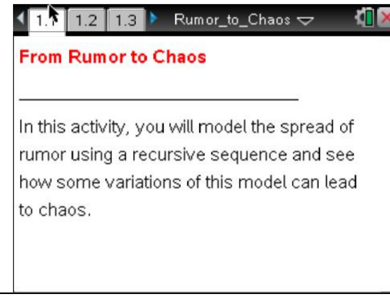
From Rumor to Chaos
Student Activity

Name ______________
Class ______________

Open the TI-Nspire document *From_Rumor_to_Chaos.tns.*

In this investigation, you will model the spread of a rumor using a recursive sequence to see how some variations of this model can lead to chaos.



Terry loves to gossip. After hearing a rumor, Terry wants all of his/her 24 friends to hear the same rumor. Terry's mother insisted that Terry can make only one phone call per day and the same phone-call restriction is true of all of Terry's friends. Assume:

A. Terry hears a rumor on day 1. On day 2, Terry randomly calls and reveals the rumor to a friend.
B. On day 3, 4, … , each person who already knows the rumor randomly calls one of the other 24 students and reveals the rumor [if that person had not already heard it].
C. The process is repeated daily until all 25 students (Terry and friends) in the class know the rumor.

Terry records the total number of people who know the rumor each day. On any given day, some people who already know the rumor might be called and told again. These people will not be counted twice. The data are given in the following table:

Day (n)	Number who knew (u_n)
1	1
2	2
3	4
4	8
5	13
6	20
7	24
8	25

Next, you will find a recursive sequence to model this data. To develop such a model, we make the assumption that:

The number of *people who learn the rumor on a given day is proportional to the product of the number of people who know the rumor and the number who don't.*

1. Explain why this assumption is reasonable.

We need to represent the following quantities in the model:

(1) the number of people who know the rumor at any given time.

(2) the number of people who do not know the rumor at any given time.

(3) the total population – or an reasonable estimate for this value.

(4) the rate at which the rumor spreads.

Let u_n denote to the number of people who know the rumor at the end of n days, P denote the total population, and k the constant of proportionality. Because of the assumption above, a reasonable recursive sequence model is

$$u_n = u_{n-1} + k \cdot u_{n-1} \cdot (25 - u_{n-1})$$

This model is an example of the general recursion model: $new\ value = old\ value + change$.

In this case, we know P = 25, so we must determine a reasonable value of k.

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

To find a reasonable value for k in this model, we first rewrite the recursive sequence as $u_n - u_{n-1} = ku_{n-1} \cdot (25 - u_{n-1})$ —an equation of the form $y = kx$ where x is the product $ku_{n-1} \cdot (25 - u_{n-1})$ and y is the difference $u_n - u_{n-1}$.

The following procedure was used to create the spreadsheet on Page 1.2. You will need to use this procedure for Problem 2 or, perhaps, on this problem if a different data set is used.

- Enter the data into Columns A and B of the table: day and num.
- Automatically calculate values for Column C (named cha) by moving your cursor to the formula cell directly under the name of the column and selecting **MENU > Data > List Operations > Difference list [ΔList] (**num**)**.
- Calculate values for Column D (named $prod$) by typing $= num \cdot (25 - num)$ in the formula cell under the name of the list.
- In Column E (named $prod2$), type $= left(prod, \dim(cha))$. This formula ensures that the lists cha and $prod2$ have the same number of elements.

Move to page 1.3.

A scatter plot using $x = prod2$, $y = cha$ has been drawn on this page.

Move to page 1.4.

2. Find a suitable value of k. Hint: Because the model has the form $y = kx$, one method is associated with linear regression.

   k = _______________

3. Using this value of k, write your model $u_n =$ _________________________________ .

Move to page 1.5.

- On this spreadsheet page, enter your answer to Question 3 as a sequence in the second cell of the Column A [model] by selecting **MENU > Data > Generate Sequence**, typing $u(n-1)$ *for* u_{n-1}, etc. and setting *initial terms* = 1 and $nMax$ = 15.

4. According to your model, how many students knew the rumor at the end of the sixth day?

5. According to your model, between which two days was the increase in the number of students knowing the rumor the largest?

Move to page 1.6.

A scatter plot of the original data is shown on Page 1.6 using x = *day* and y = *num* .

6. Draw a scatter plot of the data generated by the model using x = *day* y = *model.* How do the two scatter plots compare?

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Move to page 2.1.

Problem 2—Another Rumor Problem

In this example, Terry has more friends, but we are not sure how many. The value of P is unknown and must be estimated. The other assumptions given in the previous problem are the same. Terry recorded the data in the table below.

Day (n)	Number who knew(u_n)
1	1
2	2
3	4
4	8
5	15
6	28
7	47
8	72
9	92
10	98

7. Using this data, find a reasonable estimate for the value of P in the model.

- On Page 2.1,the data has been entered in the first two columns of the spreadsheet – day and num . Using the procedure from Question 1, create the three additional columns $cha, prod, prod2$ using the value of P from Question 7.

Move to page 2.2.

Draw a scatter plot on Page 2.2 using $x = prod2$ and $y = cha$.

Move to page 2.3.

8. Find a suitable value of k . Hint: As in Question 2, one method involves linear regression.

9. Using this value of k , write your model $u_n =$ ________________________________ .

Move to page 2.4.

- Enter your answer to Question 9 as a sequence in the first column of the spreadsheet using the process from Question 4.

10. According to your model, how many students knew the rumor at the end of the sixth day?

11. According to your model, between which two days was the increase in the number of students knowing the rumor the largest?

Move to page 2.5.

A scatter plot of the original data is shown on Page 2.5 using x = day y = num

12. Draw a scatter plot of the data generated by the model using x = day $y =$ *model*. How do the two scatter plots compare?

Move to page 3.1.

Problem 3

We will now consider a modified (logistic) model of the one used in Problems 1 and 2.

Imagine that an illness is spreading through your school. Let u_n denote the fraction of students who have the illness after n days so that $(1-u_n)$ is the fraction of the students who do not have the illness. Both u_n and $(1-u_n)$ are between 0 and 1 and the total population is now $P=1$. We have the model $u_n = ku_{n-1}(1-u_{n-1)})$ that reflects the assumption that the “”new” fraction of those having the illness, u_n, is proportional to the product of the “old” fraction who have the illness, u_{n-1}, and the fraction who do not have the illness, $(1-u_n)$. The interaction between the two groups will change the fraction representing those who have the illness at time increases.

This model is most interesting. For some values of k, the behavior of the values of u_n varies from predictable to chaotic. In this problem, we consider values of k for which 2 ≤ k ≤ 4.

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On Page 3.1, a spreadsheet has been created with two columns: num and val [value]. The entries in num are integers from 1 to 100. The entries in val are the terms in the sequence $u_n = ku_{n-1}(1-u_{n-1)})$ for values of k between 2 and 4 that you select using the clicker below the spreadsheet.

Move to page 3.2.

On Page 3.2, a scatter plot with $x = num$ x and $y = val$ has been created. The example with k = 3.2 and u_0 =.6 is shown. Consider various values for k , and toggle between Pages 3.1 and 3.2. Explore the behavior of the sequence $u_n = ku_{n-1} \cdot (1-u_{n-1})$ and the corresponding scatter plot for various values of k and various initial values u_0 where 0 < u_0 < 1. To change the value of u_0 , change the value in cell B1 on Page 3.1. The rest of the entries in this column and corresponding scatter plot on Page 3.2 are automatically updated.

13. To examine the values in the scatter plot on Page 3.2, select **MENU > Trace > Graph Trace**. There are four basic types of behavior that occur. Describe each of them, and give the approximate range of values of k for which each occurs.

