Equation of a Circle

Teacher Answers

7 8 9 **10** 11 12

TI-Nspire CAS | Investigation | Student | 120 min

Aim

The aim of this investigation is to develop the equation of a circle of radius, *r*, centred at the origin, and to explore how the equation changes when the centre is translated parallel to the *x* and *y* axes.

Equipment

For this activity you will need:

- TI-Nspire CAS
- TI-Nspire CAS document – Equation of a circle

Introduction – Setting up the calculations

Part 1 – Equation of a circle centred at the origin

This activity requires access to the "Equation of a circle" TI-Nspire document. This document should be loaded on your device before proceeding.

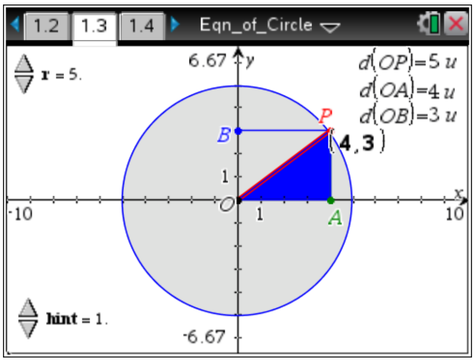
Once the document is on your handheld, press [**home**] and select **My Documents**. Locate the document and press [**enter**] to open.

Step 1 – Adjust the radius of the circle

Navigate to page **1.3** and use the spinner to adjust the radius of the circle to 5 units.

To adjust the spinner, select it by moving the cursor to the spinner and pressing [**enter**]. Then press the up and down arrows to adjust the radius. To deselect the spinner, press [**esc**].

The point ***P*** is a movable point on the circle. Points ***A*** and ***B*** mark the coordinates of *P* on the *x* and *y* axes, respectively.



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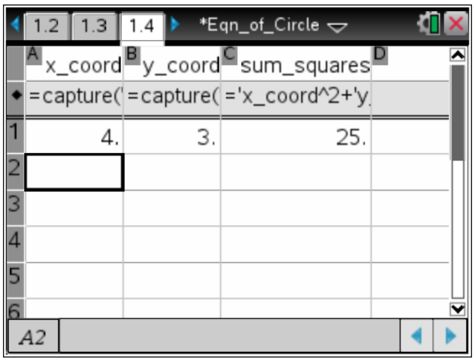
Step 2 – Observe the data capture page

Navigate to page **1.4**. The distances from the origin to *P*, *A* and *B*, d(*OP*), d(*OA*) and d(*OB*), respectively, are shown.



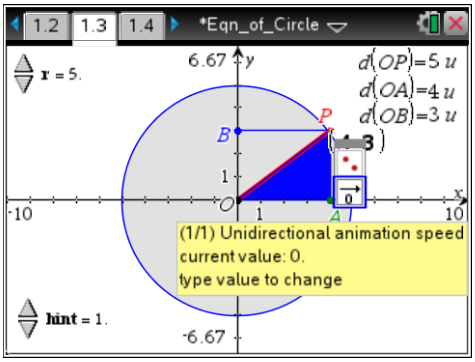
As *P* moves around the circle, the values of the *x* and *y* coordinates of *P* are captured in columns *A* and *B* on page **1.4**.

The value of $x^2 + y^2$, the sum of squares of the coordinates of *P*, is automatically calculated in column **C**.

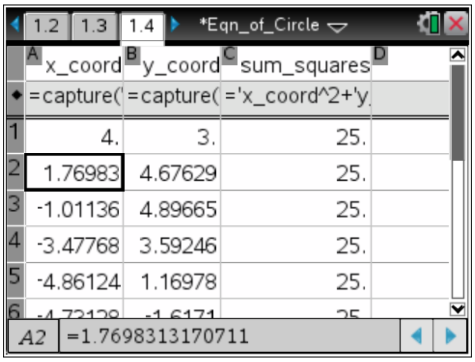
Step 3 – Animate point *P* around circle of radius 5 and capture the coordinates

Navigate back to page **1.3**. To animate the point:

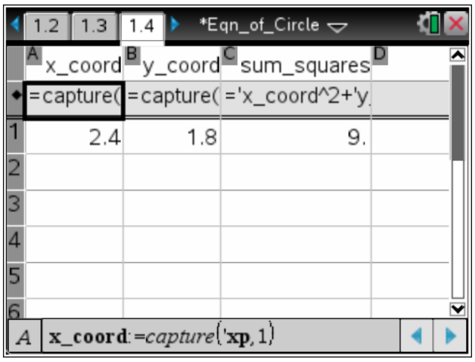
- Move the cursor to point *P* until the ‘open hand’ symbol appears.
- Press **[ctrl] > [menu]**. From the context menu that appears, select **Attributes**.
- Press the down arrow once, input the number [**9**] (fast speed) and press [**enter**].
- When point *P* has moved through a complete revolution around the circle, press [**esc**] to stop the animation.



Navigate back to page **1.4**. The *x* and *y* coordinates will be captured and $x^2 + y^2$ calculated.



Later in the activity, you will be asked to clear this collected data. This will enable you to collect new data for circles with different radii. To clear the collected data, move the cursor into each of the ‘**capture**’ or ‘**coord**’ cells in the second row and press [**enter**] twice. If you see any messages or warnings, select **OK**.



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Questions

1. What do you notice about the value of the sum of squares, $x^2 + y^2$, irrespective of the coordinates of *P*?
When the radius of the circle is 5, the value of the sum of squares of the coordinates of *P*, ($x^2 + y^2$) is equal to 25 in all cases.

2. Explain why the value of the sum of squares ($x^2 + y^2$) is unchanged despite the *x* and *y* coordinates of *P* changing (Hint: look at ΔOPA and ΔOPB).
Triangles *OPA* and *OPB* are right-angled triangles with hypotenuse *OP*. By Pythagoras' Theorem, the square of the hypotenuse, $(\text{d}(OP))^2$, is always equal to $(\text{d}(OA))^2 + (\text{d}(OP))^2 = x^2 + y^2$.

3. Use your findings from Questions 1 and 2 above to complete the equation of a circle of radius 5:
$x^2 + y^2 = 25$

Step 4 – Change the radius of the circle and capture the *x* and *y* coordinates of *P*

- Use the spinner on page **1.3** to change the radius of the circle to a value of your choosing.
- Navigate to page **1.4** and reset the data in columns **A**, **B** and **C**.

Step 5 – Animate *P* around the circle of your chosen radius and capture the coordinates

- Use the method described in **Step 3** above to animate the point ***P***.
- Navigate to page **1.4** and observe the value of $x^2 + y^2$ in the cells of column **C**.

Repeat Steps 4 and 5 above for three other values of *r*.

4. For the values of *r* that you investigated, complete the table for the equations of the circles.

Radius	Sum of squares of the coordinates of *P* (column C of page 1.4)	Equation of circle
$r = 5$	$(\text{d}(OA))^2 + (\text{d}(OB))^2 = 25$	$x^2 + y^2 = 25$
The answers below will vary, depending of the values of r chosen by the student. All answers will be of the form $x^2 + y^2 = r^2$.		
$r = 2$	$(\text{d}(OA))^2 + (\text{d}(OB))^2 = 4$	$x^2 + y^2 = 4$
$r = 3$	$(\text{d}(OA))^2 + (\text{d}(OB))^2 = 9$	$x^2 + y^2 = 9$
$r = 4$	$(\text{d}(OA))^2 + (\text{d}(OB))^2 = 16$	$x^2 + y^2 = 16$
$r = 6$	$(\text{d}(OA))^2 + (\text{d}(OB))^2 = 36$	$x^2 + y^2 = 36$

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5. Use what you have learnt from completing the table above to write down the radius, *r*, of the circles with the following equations.

(a) Equation: $x^2 + y^2 = 100$, $r = 10$

(b) Equation: $x^2 + y^2 = 81$, $r = 9$

(c) Equation: $x^2 + y^2 = 1$, $r = 1$

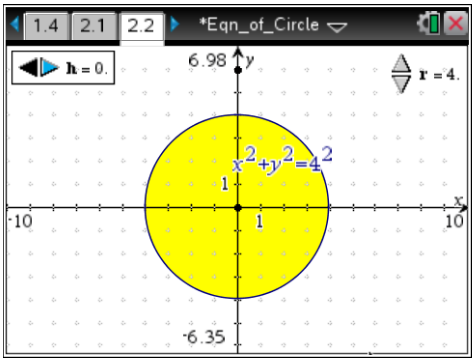
(d) Equation: $x^2 + y^2 = 2$, $r = \sqrt{2}$

(e) Equation: $x^2 + y^2 = 10$, $r = \sqrt{10}$

(f) Equation: $x^2 + y^2 = \frac{1}{4}$, $r = \frac{1}{2}$

Part 2 – Equation of a circle translated parallel to the *x*-axis

Navigate to page **2.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to 4 units. The equation of the circle is displayed as $x^2 + y^2 = 4^2$.



Use the spinner labelled **h** to translate an image of the circle to the left and to the right. Note the equation and coordinates of the centre of the translated circle.

6. Use your observations about the effect of changing the value of *h* to complete the table below.

Radius	Translation parallel to *x*-axis	Coordinates of centre	Equation of circle
$r = 4$	$h = 0$	(0, 0)	$x^2 + y^2 = 16$
$r = 4$	$h = 1$	(1, 0)	$(x - 1)^2 + y^2 = 16$
$r = 4$	$h = 2$	(2, 0)	$(x - 2)^2 + y^2 = 16$
$r = 4$	$h = 5$	(5, 0)	$(x - 5)^2 + y^2 = 16$
$r = 4$	$h = 9$	(9, 0)	$(x - 9)^2 + y^2 = 16$
$r = 4$	$h = -1$	(–1, 0)	$(x + 1)^2 + y^2 = 16$
$r = 4$	$h = -2$	(–2, 0)	$(x + 2)^2 + y^2 = 16$
$r = 4$	$h = -4$	(–4, 0)	$(x + 4)^2 + y^2 = 16$
$r = 4$	$h = -6$	(–6, 0)	$(x + 6)^2 + y^2 = 16$
$r = 4$	$h = -8$	(–8, 0)	$(x + 8)^2 + y^2 = 16$

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7. Explore other values of r and h and record the results in the table below.

Radius	**Translation**	**Centre**	**Equation of circle**
Results will depend on the values of r and h chosen. The centre will be of the form $(h, 0)$ and the equation of the circle of the form $(x-h)^2 + y^2 = r^2$			
$r = 2$	$h = 4$	$(4, 0)$	$(x-4)^2 + y^2 = 4$
$r = 3$	$h = 2$	$(2, 0)$	$(x-2)^2 + y^2 = 9$
$r = 5$	$h = -5$	$(-5, 0)$	$(x+5)^2 + y^2 = 25$
$r = 6$	$h = -1$	$(-1, 0)$	$(x+1)^2 + y^2 = 36$

8. Use what you have learnt from completing the tables above to write down the radius, r, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $(x-2.5)^2 + y^2 = 64$, $r = 8$ centre $(2.5, 0)$

(b) Equation: $\left(x+\frac{1}{2}\right)^2 + y^2 = 400$, $r = 20$ centre $\left(\frac{1}{2}, 0\right)$

(c) Equation: $(x+5)^2 + y^2 = 6$, $r = \sqrt{6}$ centre $(-5, 0)$

(d) Equation: $\left(x-\frac{8}{3}\right)^2 + y^2 = 36$, $r = 6$ centre $\left(\frac{8}{3}, 0\right)$

(e) Equation: $(x-12)^2 + y^2 = 1$, $r = 1$ centre $(12, 0)$

(f) Equation: $(x+8.3)^2 + y^2 = 14$, $r = \sqrt{14}$ centre $(8.3, 0)$

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9. Use what you have learnt to write the equations of the circles with the following radii and centres.

(a) Radius: $r = 9$, centre: $(11, 0)$ Equation $(x-11)^2 + y^2 = 81$

(b) Radius: $r = 1$, centre: $\left(-\frac{1}{3}, 0\right)$ Equation $\left(x+\frac{1}{3}\right)^2 + y^2 = 1$

(c) Radius: $r = 7$, centre: $(3.4, 0)$ Equation $(x-3.4)^2 + y^2 = 49$

(d) Radius: $r = \sqrt{5}$, centre: $(-6, 0)$ Equation $(x+6)^2 + y^2 = 5$

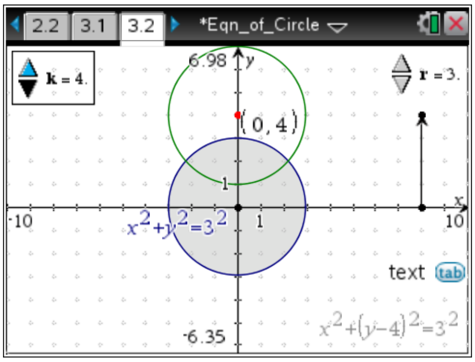
(e) Radius: $r = 16$, centre: $\left(\frac{12}{5}, 0\right)$ Equation $\left(x-\frac{12}{5}\right)^2 + y^2 = 256$

(f) Radius: $r = 2\sqrt{3}$, centre: $(-10, 0)$ Equation $(x+10)^2 + y^2 = 12$

(Note: $\left(2\sqrt{3}\right)^2 = 2^2 \times \left(\sqrt{3}\right)^2 = 12$)

Part 3 – Equation of a circle translated parallel to the *y*-axis

Navigate to page **3.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to 3 units. The equation of the circle is displayed as $x^2 + y^2 = 3^2$.



Use the spinner labelled **k** to translate an image of the circle to up and down. Note the equation and coordinates of the centre of the translated circle.

10. Use your observations about the effect of changing the value of *k* to complete the table below.

Radius	Translation parallel to *y*-axis	Coordinates of centre	Equation of circle
$r = 3$	$k = 0$	(0, 0)	$x^2 + y^2 = 9$
$r = 3$	$k = 1$	(0, 1)	$x^2 + (y-1)^2 = 9$
$r = 3$	$k = 3$	(0, 3)	$x^2 + (y-3)^2 = 9$
$r = 3$	$k = 5$	(0, 5)	$x^2 + (y-5)^2 = 9$
$r = 3$	$k = -2$	(0, –2)	$x^2 + (y+2)^2 = 9$
$r = 3$	$k = -4$	(0, –4)	$x^2 + (y+4)^2 = 9$
$r = 3$	$k = -6$	(0, –6)	$x^2 + (y+6)^2 = 9$

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11. Explore other values of r and k and record the results in the table below.

Radius	Translation	Centre	Equation of circle
Results will depend on the values of r and k chosen. The centre will be of the form $(0, k)$ and the equation of the circle of the form $x^2 + (y - k)^2 = r^2$			
$r = 2$	$k = 3$	$(0, 3)$	$x^2 + (y - 3)^2 = 4$
$r = 4$	$k = 5$	$(0, 5)$	$x^2 + (y - 5)^2 = 16$
$r = 5$	$k = -2$	$(0, -2)$	$x^2 + (y + 2)^2 = 25$
$r = 6$	$k = -6$	$(0, -6)$	$x^2 + (y + 6)^2 = 36$

12. Use what you have learnt from completing the tables above to write down the radius, r, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $x^2 + \left(y + \frac{3}{4}\right)^2 = 1$, $r = 1$ centre $\left(0, -\frac{3}{4}\right)$

(b) Equation: $x^2 + \left(y - 5.3\right)^2 = 49$, $r = 7$ centre $(0, 5.3)$

(c) Equation: $x^2 + \left(y - 9\right)^2 = 7$, $r = \sqrt{7}$ centre $(0, 9)$

(d) Equation: $x^2 + \left(y + 5\right)^2 = \frac{16}{9}$, $r = \frac{4}{3}$ centre $(0, -5)$

13. Use what you have learnt to write the equations of the circles with the following radii and centres.

(a) Radius: $r = 4$, centre: $(0, -3)$ Equation $x^2 + (y + 3)^2 = 16$

(b) Radius: $r = 1$, centre: $\left(0, \frac{2}{3}\right)$ Equation $x^2 + \left(y - \frac{2}{3}\right)^2 = 1$

(c) Radius: $r = \sqrt{3}$, centre: $(0, 1.5)$ Equation $x^2 + (y - 1.5)^2 = 3$

(d) Radius: $r = \frac{5}{2}$, centre: $\left(0, -\frac{1}{2}\right)$ Equation $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$

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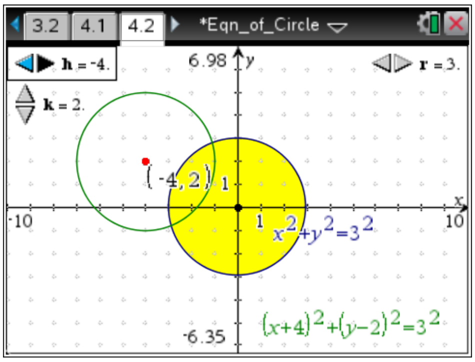
Part 4 – Equation of a circle with centre at any point on the plane

Navigate to page **4.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to the value specified in the table below.

Use the spinners labelled **h** and **k** to translate the centre of an image of the circle to the coordinates specified in the table below.



14. Note the equation of the translated circle and of the value of **h** and **k** that translate the centre of the circle from the origin to the desired point on the plane. Complete the table below.

Radius	Coordinates of the centre	Value of *h*	Value of *k*	Equation of circle
$r = 3$	(2, 1)	2	1	$(x-2)^2 + (y-1)^2 = 9$
$r = 4$	(1, 2)	1	2	$(x-1)^2 + (y-2)^2 = 16$
$r = 5$	(−3, 2)	−3	2	$(x+3)^2 + (y-2)^2 = 25$
$r = 3$	(3, −2)	3	−2	$(x-3)^2 + (y+2)^2 = 9$
$r = 2$	(−6, −2)	−6	−2	$(x+6)^2 + (y+2)^2 = 4$
$r = 4$	(6, 2)	6	2	$(x-6)^2 + (y-2)^2 = 16$
$r = 3$	(−5, −4)	−5	−4	$(x+5)^2 + (y+4)^2 = 9$
$r = 2$	(−5, 3)	−5	3	$(x+5)^2 + (y-3)^2 = 4$
$r = 1$	(5, −3)	5	−3	$(x-5)^2 + (y+3)^2 = 1$
$r = 5$	(−4, −1)	−4	−1	$(x+4)^2 + (y+1)^2 = 25$

15. Use what you have learnt from completing the table above to write down the radius, **r**, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $(x-3.5)^2+(y+1.2)^2=1$ $\quad r=1$ $\quad$ centre (3.5, −1.2)

(b) Equation: $(x+2.7)^2+(y+3.1)^2=64$ $\quad r=8$ $\quad$ centre (−2.7, −3.1)

(c) Equation: $\left(x-\frac{7}{8}\right)^2+\left(y-\frac{9}{10}\right)^2=144$ $\quad r=12$ $\quad$ centre $\left(\frac{7}{8},\frac{9}{10}\right)$

(d) Equation: $(x+6.4)^2+(y-5.1)^2=\frac{25}{4}$ $\quad r=\frac{5}{2}$ $\quad$ centre (−6.4, 5.1)

(e) Equation: $\left(x-\frac{5}{6}\right)^2+(y+3)^2=6$ $\quad r=\sqrt{6}$ $\quad$ centre $\left(\frac{5}{6},-3\right)$

(f) Equation: $(x+0.7)^2+(y+1.3)^2=2.25$ $\quad r=1.5$ $\quad$ centre (−0.7, −1.3)

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