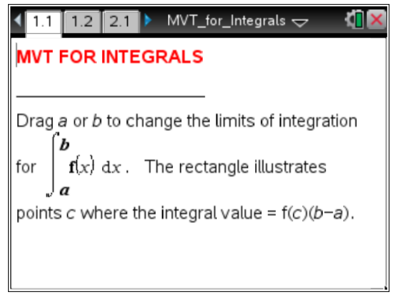
MVT for Integrals

Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *MVT_for_Integrals.tns*.



The Mean Value Theorem for Integrals states that if $\mathbf{f}(x)$ is continuous on the interval $[a, b]$, then there exists a number c in the interval (a, b) such that: $\int_a^b \mathbf{f}(x)dx = \mathbf{f}(c)(b-a)$.

In this activity, you will explore a visual representation of this theorem and consider some of its implications.

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1. The graph displays the graph of $y = \mathbf{f}(x)$ on the closed interval $[a, b]$. Drag endpoints a and b along the x-axis to change the interval.
   a. Describe the relationship between the shaded region and the definite integral calculation below the graph.

   b. Place endpoint a at 1 and describe the relationships between the shaded region and the displayed rectangle as you move the other endpoint, b, to the right.

   c. Make a conjecture about the relationship between the area of the rectangle and the definite integral over the same interval. Explain how you might justify your conjecture.

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2. The graph shown is of the same function as the previous page, with the area of the rectangle now displayed in the upper right corner. Use the graph to calculate each integral and describe how these values relate to the area of the rectangle in each case:

   a. $\int_0^3 \mathbf{f}(x)dx$ b. $\int_1^4 \mathbf{f}(x)dx$

   c. $\int_0^6 \mathbf{f}(x)dx$ d. $\int_2^7 \mathbf{f}(x)dx$

3. One side of the rectangle is parallel to the *x*-axis and intersects the graph of $y = \mathbf{f}(x)$ at the point given by coordinates $(c, \mathbf{f}(c))$.
   a. Explain how the area of this rectangle is represented in the statement of the Mean Value Theorem for Integrals given on the previous page.

   b. The average (or mean) value of a function **f** over a closed interval $[a, b]$ is defined as $\dfrac{\int_a^b \mathbf{f}(x)dx}{b-a}$.

   The title "Mean Value Theorem for Integrals" suggests that it has something to do with mean value. Explain how the point $(c, \mathbf{f}(c))$ relates to mean value.

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4. Drag *a* and/or *b* to identify intervals for which the definite integral for this new function $\mathbf{f}(x)$ is positive and for which it is negative.
   a. What do you notice about the rectangle in each case?

   b. What does this tell you about the average value of the function over these intervals? Explain.

5. Suppose the average value of the function $\mathbf{f}(x)$ is zero.
   a. What would the rectangle look like in this case?

   b. What could you say about the definite integral and the area bounded by $\mathbf{f}(x)$ and the *x*-axis in this case?

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6. Explain how you could use the graph to find the average value of this new function over the interval $[-1, 2]$.

7. Is there an x-value in the interval [–1, 2] for which the function value is equal to this average value? Explain how you can determine this.

8. Repeat questions 6 and 7 for the interval [–3, 0].

9. Both the Mean Value Theorem and the Mean Value Theorem for Integrals are known as “existence” theorems. Explain what this means in each case.

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10. Use the graph to find the average value of the greatest integer function on the following intervals and explain how you determined each of these values:
    a. [–1, 4]

    b. [2, 4]

    c. [–1, 3]

11. Which of these average values are possible function values? Explain.

12. Does this example violate the Mean Value Theorem for Integrals? Explain why or why not.