

# Probable Factors

## Teacher Notes & Answers

7 8 9 10 11 12



### Introduction

What is the probability that a randomly generated quadratic can be factorised? This investigation looks at a significantly reduced set of quadratics using dice to generate the coefficients. The coefficient of  $x^2$  is 1. The remaining coefficients are generated randomly using dice. Two dice are rolled; the numbers appearing uppermost are added together and form the  $x$  coefficient ( $b$ ) in the quadratic:  $y = x^2 + bx + c$ . Another two dice are rolled; the numbers appearing uppermost multiply together and form the constant ( $c$ ). What is the probability this quadratic can be factorised?

- Over the rational number field?
- Over the real number field?

### Teacher Notes:

Before commencing the digital simulation, it is important for students to generate a number of quadratics by hand using regular dice. This helps students understand what the simulation is doing and establishes a clear focus for the investigation.

### Demonstration:

Roll two dice and write the results on the board for students to see, including the first part of the quadratic.

Example: (3, 5)    Coefficient:  $3 + 5 = 8$     Quadratic:  $x^2 + 8x +$

Roll the second pair of dice, write the results on the board and complete the quadratic:

Example: (6, 2)    Constant:  $6 \times 2 = 12$     Quadratic:  $x^2 + 8x + 12$

Does this quadratic factorise?

Example:  $x^2 + 8x + 12 = (x + 6)(x + 2)$

This example represents 'success' as the randomly produced quadratic factorised.

Repeat this process, rolling new numbers for the coefficient and the constant.

Example: (4, 1)    Coefficient:  $4 + 1 = 5$     Quadratic:  $x^2 + 5x +$

Roll the second pair of dice, write the results on the board and complete the quadratic:

Example: (5, 2)    Constant:  $5 \times 2 = 10$     Quadratic:  $x^2 + 5x + 10$

Does this quadratic factorise?

Example:  $x^2 + 5x + 10 =$  *Does not factorise*

This example represents 'failure' as the randomly produced quadratic did not factorise.

Students should now try one for themselves. Ask students to volunteer their randomly generated quadratics and write them on the board. How many of the quadratics factorise? What is the probability that quadratics produced using this process will factorise over the rational number field? (This is the focus for the investigation.)

The quantity of calculations required to determine the theoretical probability is quite significant. It is therefore desirable to establish an estimate using a simulation. The simulation can be achieved using the calculator and the instructions provided. It is possible to produce the document and send it to students rather than have each student create the simulation document independently. This choice is left to the individual and depends on how much time is to be devoted to learning how to use the calculator as an exploratory tool and how much is to focus specifically on identifying and answer from the simulation.

The simulation generates the equations. It is possible to generate them all in a spreadsheet using a countif function, however, seeing the equations is part of the development of key concepts.

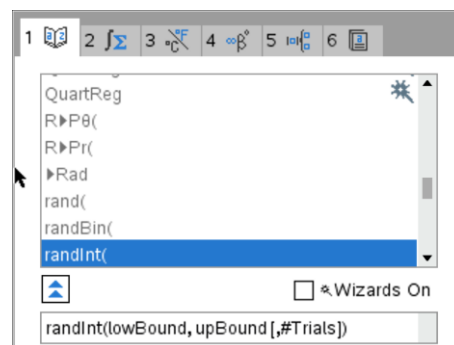
### Introduction – Setting up the simulation

Start a new document and insert a spreadsheet. The spreadsheet will be used to generate the random integers (whole numbers) in place of a die.

In cell A1 type the formula: =randInt(1,6)

This produces a random integer between 1 and 6.

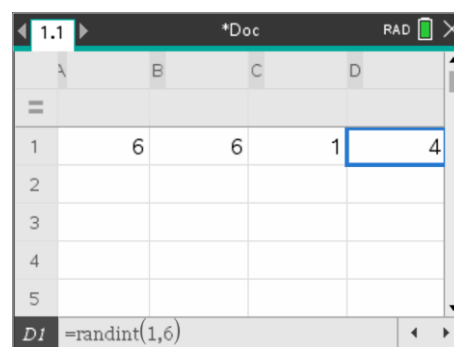
**Note:** In the spreadsheet application the fastest way to access the **randInt()** command is through the catalogue.



The formula in cell A1 can be copied into cells B1, C1 and D1.

Click on cell A1, press **CTRL + C** to copy, move to cell B1 and press **CTRL + V** to paste.

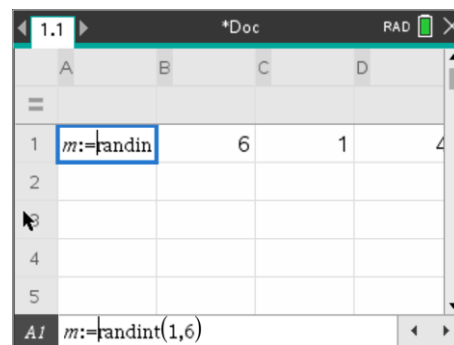
Repeat this process for cells C1 and D1.



The simulated dice rolls need to be stored. Select cell A1 and press **CTRL + VAR** and store the roll in 'm'.

Repeat this for cell B1, store as 'n'; store cell C1 as 'p' and cell D1 as 'q'.

**Note:** The simulated dice rolls become bold once they have been stored. Press **CTRL + R** to simulate dice rolls and observe how each cell contains a new outcome each time CTRL + R is pressed.

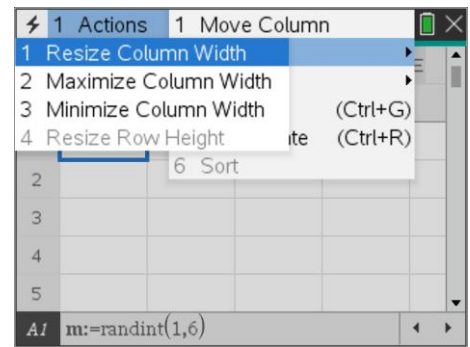




The column width can be adjusted using the Actions menu.

[ Menu ] > Actions > Resize > Resize Column Width

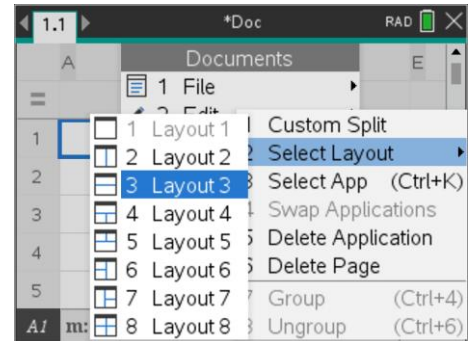
Use the left / right arrow keys to change the column width.



The next step is to split the current page into two applications.

[ Doc ] > Page Layout > Layout 3

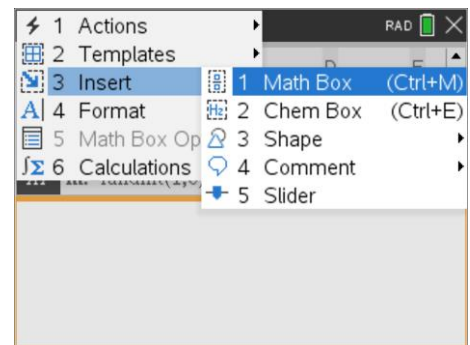
This will split the current page into two horizontal regions.



As the page is split into two you will be prompted to insert another application. Insert a **Notes** Application.



On the calculator, Ctrl + Tab performs a similar role to Alt + Tab in Microsoft Windows™, this shifts the focus between active applications.



The first step is to define a quadratic function.

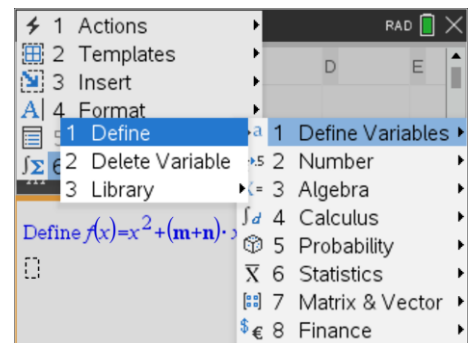
Press **Ctrl + M** to insert a Maths Box into the Notes Application.

In the Maths box define the function  $f(x) = x^2 + (m+n) \cdot x + p \cdot q$

[ Menu ] > Calculations > Define Variables > Define

A multiplication sign must be used between the parenthesis and the x, similarly between the p and q to ensure these operations are multiplied.

Notice the defined variables m, n, p and q appear bold.

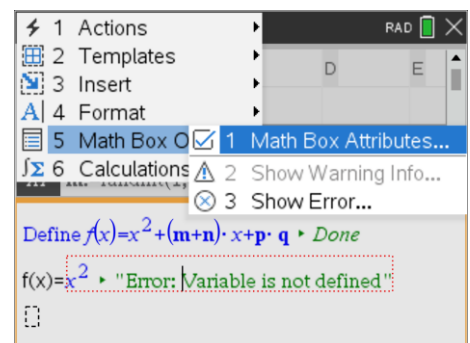


The TI-Nspire non-CAS platform does not support algebraic functionality in any form, some clever use of text and Maths boxes can be used to display the equation.

**Text:** f(x) =

**Maths Box:**  $x^2$

When  $x^2$  is entered into the maths box an error message will be displayed. Accept the error message and change the Maths Box Attributes through the Maths Box Options. In the input/output option, select Hide Output (error message). The result will be the expression  $x^2$  in the maths-box. Move the cursor out of the Maths box to observe the change.



This process will be repeated for the quadratic function.

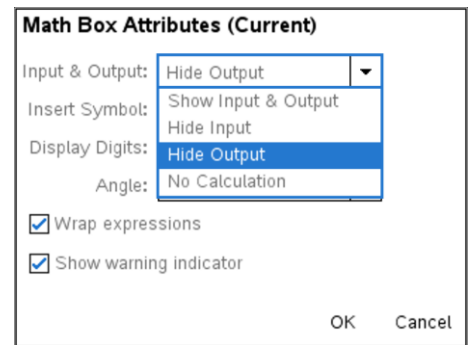
**Text:** +

**Maths Box:**  $m + n$

This time, hide the input.

**Maths Box:**  $x$

Hide the output (error).



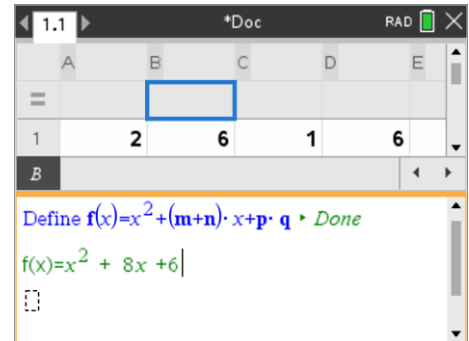
Almost done ...

**Text:** +

**Maths Box:**  $p \times q$

Hide the input.

You can change the colour of the text to give the equation some consistency.

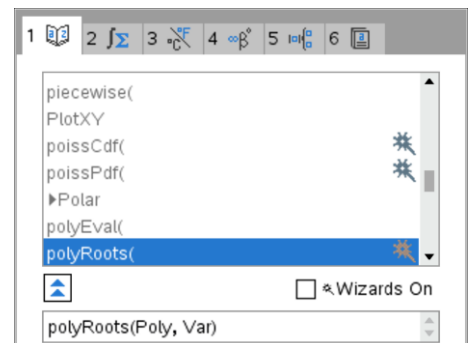


The final step in this special simulator is to find the roots.

In a maths box, press the [ Catalogue ] key:

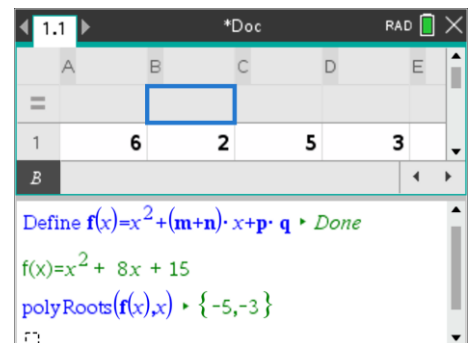
Press P and navigate to the Poly Roots command. Enter the equation definition and variable.

$$\text{PolyRoots}(f(x), x)$$



Navigate back to the spreadsheet (**Ctrl + Tab**) and press **Ctrl + R** to 'roll' the dice. Notice what happens to the quadratic equation!

In the example shown opposite:  $f(x) = x^2 + 8x + 15$  has two roots since  $f(x) = (x + 5)(x + 3)$  giving the roots as:  $\{-5, -3\}$ .



### Probability through simulation

1. Press **Ctrl + R** to simulate several rolls. How do the roots of the polynomial indicate whether the quadratic factorises over:
  - a. Rational numbers. **Answer:** Whole numbers displayed as a set  $\{\#, \#\}$  or  $\{\#\}$  for perfect squares.
  - b. Real numbers. **Answer:** Decimal approximations, displayed as above.
  - c. Doesn't factorise? **Answer:** Null set  $\{\}$ .

2. To get a sense of the probability of rolling a quadratic that will factorise over the rational number field, simulate 100 rolls of the dice. Watch the notes page and count how many of the quadratic equations have rational factors. Write your answer as a probability or percentage.

**Answer:** Answers will vary typically between 7% and 27% with the actual probability  $\approx 17.4\%$ .

3. Your response to question two provides an indication of the theoretical answer, how could a better estimate of the theoretical probability be obtained?

**Answer:** More simulations could provide a better estimate. Students may also suggest 'averaging' class results or 'plotting' class results.

**Teacher Notes:** If students suggesting 'averaging' class results, consider using either TI-Navigator or 'sticky dots' to generate a dot plot of the distribution of student results. Whilst a class of 25 students doesn't generate sufficient data to show that the data is 'normally distributed' (Central Limit Theorem), it provides students some understanding of a sampling distribution! The TI-Nspire file "Probable Factors" contains a program in Problem 2 titled: ratsim(). Run this program to generate multiple quadratics in multiple samples. More details are provided at the end of this document.

4. Complete the lattice diagram below for the sum of the two dice; some of the answers have been completed. The lattice diagram represents all the possible coefficients for  $x$ , in the general form:  $y = x^2 + bx + c$

Dice Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

5. Complete the lattice diagram below for the product of the two dice; some of the answers have already been filled in. The lattice diagram represents all the possible constants in  $y = x^2 + bx + c$

Dice Prod.	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The two lattice diagrams show the individual possibilities for the coefficient ( $b$ ) and the constant ( $c$ ). For each coefficient there are 36 permutations making a total of  $36 \times 36 = 1296$  possible equations to be factorised. Some of the equations are repeated, systematic exploration combined with some thinking and reflection will drastically reduce the time taken to explore which of these 1296 combinations factorise.

6. Suppose a [1] and a [1] are rolled on the first pair of dice to give an x coefficient of 2.  
 The equation would be:  $y = x^2 + 2x + c$  Two more dice must be rolled, the product of which form the constant (c).  
 The lattice diagram for the product can be used to write down all the possible quadratics:  $y = x^2 + 2x + 1$ ,  
 $y = x^2 + 2x + 2 \dots y = x^2 + 2x + 36$

a. Write down all the quadratics that can be formed.

- |                     |                     |
|---------------------|---------------------|
| $y = x^2 + 2x + 1$  | $y = x^2 + 2x + 2$  |
| $y = x^2 + 2x + 3$  | $y = x^2 + 2x + 4$  |
| $y = x^2 + 2x + 5$  | $y = x^2 + 2x + 6$  |
| $y = x^2 + 2x + 8$  | $y = x^2 + 2x + 9$  |
| $y = x^2 + 2x + 10$ | $y = x^2 + 2x + 12$ |
| $y = x^2 + 2x + 15$ | $y = x^2 + 2x + 16$ |
| $y = x^2 + 2x + 18$ | $y = x^2 + 2x + 20$ |
| $y = x^2 + 2x + 24$ | $y = x^2 + 2x + 25$ |
| $y = x^2 + 2x + 30$ | $y = x^2 + 2x + 36$ |

b. Determine which quadratics can be factorised.

$y = x^2 + 2x + 1$        $(x + 1)^2$

c. Graph each of the equations indicating which ones factorised successfully over the rational number field.

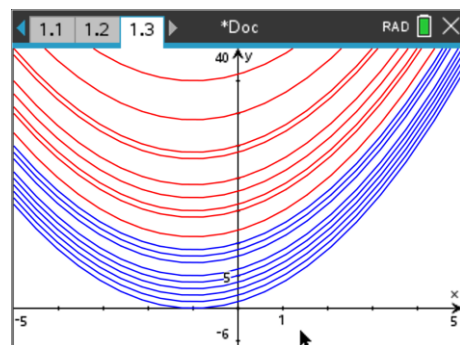
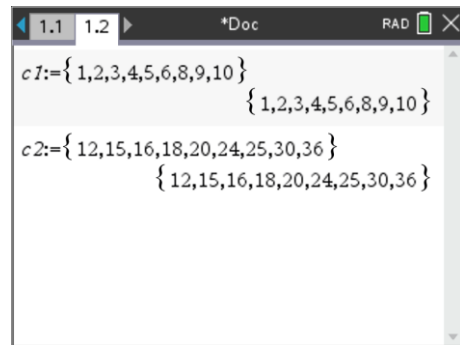
A fast way to produce the family of curves is to set  $c := \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12 \dots 36\}$ .

There is a limit of 16 values for a parameter such as c, when graphing a family of curves such as:  $x^2 + 2x + c$ . There are 18 elements in our value for c. A quick way to get around this problem is to have C1 and C2.

C1:= $\{1, 2, 3, 4, 5, 6, 8, 9, 10\}$   
 C2: =  $\{12, 15, 16, 18, 20, 24, 25, 30, 36\}$

The only equation that factorises is:  
 $y = x^2 + 2x + 1$

This is a perfect square and is the only graph that touches the x axis.



d. Use the lattice diagram for the **dice products** to determine how many of the quadratics of the form:  $y = x^2 + 2x + c$  will factorise.

The dice products lattice shows that only one combination generates a quadratic that factorises to produce a constant  $c = 1$ . :  $\{1, 1\}$

7. Suppose a [5] and a [3] are rolled on the first pair of dice to give an x coefficient of 8.

The equation would be:  $y = x^2 + 8x + c$  Two more dice must be rolled, the product of which form the constant

(c). The lattice diagram for the product can be used to write down all the possible quadratics:  $y = x^2 + 8x + 1$ ,  
 $y = x^2 + 8x + 2 \dots y = x^2 + 8x + 36$

- a. Write down all the quadratics that can be formed.

$$y = x^2 + 8x + 1 \quad y = x^2 + 8x + 2$$

$$y = x^2 + 8x + 3 \quad y = x^2 + 8x + 4$$

$$y = x^2 + 8x + 5 \quad y = x^2 + 8x + 6$$

$$y = x^2 + 8x + 8 \quad y = x^2 + 8x + 9$$

$$y = x^2 + 8x + 10 \quad y = x^2 + 8x + 12$$

$$y = x^2 + 8x + 15 \quad y = x^2 + 8x + 16$$

$$y = x^2 + 8x + 18 \quad y = x^2 + 8x + 20$$

$$y = x^2 + 8x + 24 \quad y = x^2 + 8x + 25$$

$$y = x^2 + 8x + 30 \quad y = x^2 + 8x + 36$$

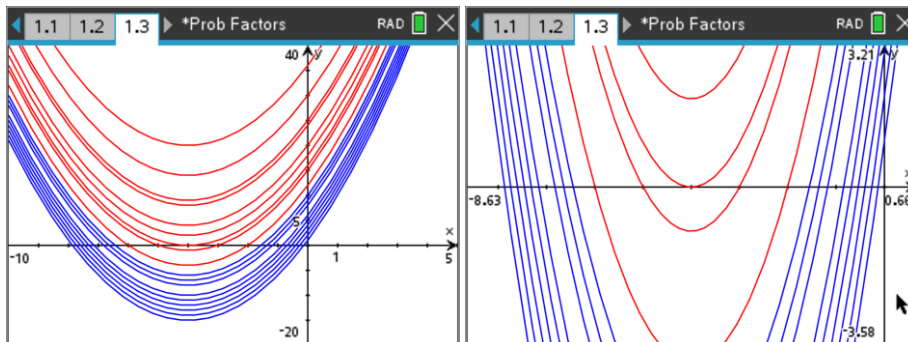
- b. Determine which quadratics can be factorised.

$$y = x^2 + 8x + 12 \quad y = (x + 6)(x + 2)$$

$$y = x^2 + 8x + 15 \quad y = (x + 5)(x + 3)$$

$$y = x^2 + 8x + 16 \quad y = (x + 4)^2$$

- c. Graph each of the equations indicating which ones factorised successfully over the rational number field.



**Teacher Note:** Students can use the same  $C_1$  and  $C_2$  used in the previous problem to graph the family of quadratics. Note that there are many quadratics that factorise over the real numbers, but only a subset that have rational factors. Students should be encouraged to use the zoom feature to get a closer view of the axis intercept. Recall also that Ctrl + Z will undo a recent change, including zooming in. So once zoomed in, students can use Ctrl + Z to return to the previous window setting.

- d. Consider both sets of graphs  $y = x^2 + 2x + c$  and  $y = x^2 + 8x + c$  and observe the value for the constant where the family of graphs no longer cross the x axis.
- i. Comment on your findings.

**Answer:** When  $b = 2$ , the graphs only cross the axis when  $c = 1$ . When  $b = 8$ , the graphs only cross the axis when  $c = 16$ . Comments may vary, some students may identify that both 1 and 16 are perfect squares and perhaps that  $c = \left(\frac{b}{2}\right)^2$



For equations:  $y = x^2 + 8x + c$  – detailed analysis

- There are 6 equations that do not cross the x axis and therefore do not factorise over the real number field.
  - There are 12 equations that cross or touch the x axis but only 3 of them have rational x intercepts and therefore rational factors.
  - As per the previous set of equations, it is the 'perfect square' that is the 'last' of the sequence of graphs to cross the x axis.
  - It is the 'perfect square' concept that allows students to progress rapidly through the remainder of the activity. It reduces enormously the number of equations that need to be investigated for factorising. It also yields a simple method for investigating factorising over the real number field.
- ii. Use this to predict or hypothesise the value of the constant (c) such that the family of graphs  $y = x^2 + 10x + c$  will no longer cross the x axis.

**Answer:** Student responses may vary, depending on their conclusions from part (i). For those that correctly identified the pattern they should determine that  $c = 25$ .

**Teacher Note:** The purpose of asking students to 'predict' very early in the investigation is to help them realise that they generally need more data to draw any conclusions. If students get stuck on this question, ask them "what could help you identify a pattern?" The formal questions may not require more exploration, however there is no reason why, particularly with the accessible technology, students cannot explore further and summarise their results.

- iii. Check your hypothesis with your teacher and discuss how this can reduce the number of possible equations to be explored in this investigation.

It is important for students to notice the 'perfect square' at this point. If students have not yet identified the relationship, draw the family of graphs for:

$$y = x^2 + 10x + c$$

Followed by:

$$y = x^2 + 12x + c$$

Students should write a description for identifying the value of c given the x coefficient. (half and square) This reduces the number of equations that need to be explored from the dice product lattice diagram

- iv. Use the "dice product" lattice diagram to determine how many of the quadratic equations with an x coefficient of 8 will factorise?

If  $c = 12, 15$  or  $16$  the quadratic can be factorised.

12 can be produced with products:  $\{6, 2\}$  and  $\{3, 4\}$ .

15 can be produced by  $\{3, 5\}$ .

16 can be produced by  $\{4, 4\}$ .

- e. What other dice combinations are possible to produce an x coefficient of 8?

The dice combinations provided in the question was  $[5, 3]$ . Other dice combinations to produce an x coefficient of 8 include:  $[6, 2]$  and  $[4, 4]$ ; however students may also include  $[2, 6]$  and  $[3, 5]$

The purpose of the question is to have students realise that upon determining the number of successful outcomes for  $y = x^2 + 8x + c$  they can complete more than just the  $[5, 3]$  combination in the lattice diagram.



8. The modified lattice diagram below includes a space to record the number of quadratics that factorise for the corresponding  $x$  coefficient. Complete the lattice diagram with each of the associated results.

Complete Analysis:

If the quadratic equation is:  $x^2 + 2x + c$  rational factors will only occur when  $c = 1$ . Checking the dice product lattice diagram this will only occur if (1, 1) is rolled. The result for this quadratic equation is 1 occurrence.

If the quadratic equation is:  $x^2 + 3x + c$  rational factors will only occur when  $c = 2$ . Checking the dice product lattice diagram this will only occur if a (1, 2) or (2, 1) combination is rolled. The result for this quadratic equation is 2 occurrences.

If the quadratic equation is:  $x^2 + 4x + c$  rational factors will only occur when  $c = 3$  or 4. Checking the dice product lattice diagram a product of 3 or 4 can be produced by any of the following dice combinations: (1, 3), (3, 1), (2, 2), (1, 4) or (4, 1). The result for this quadratic equation is 5 occurrences.

If the quadratic equation is:  $x^2 + 5x + c$  rational factors will only occur when  $c = 4$  or 6. Checking the dice product lattice diagram a product of 4 or 6 can be produced by any of the following dice combinations: (1, 4), (4, 1), (2, 2), (1, 6), (6, 1), (2, 3) or (3, 2). The result for this quadratic equation is 7 occurrences.

If the quadratic equation is:  $x^2 + 6x + c$  rational factors will only occur when  $c = 5, 8$  or 9. Checking the dice product lattice diagram a product of 5, 8 or 9 can be produced by any of the following dice combinations: (1, 5), (5, 1), (2, 4), (4, 2) or (3, 3). The result for this quadratic equation is 5 occurrences.

If the quadratic equation is:  $x^2 + 7x + c$  rational factors will only occur when  $c = 6, 10$  or 12. Checking the dice product lattice diagram a product of 6, 10 or 12 can be produced by any of the following dice combinations: (1, 6), (6, 1), (2, 3), (3, 2), (2, 5), (5, 2), (2, 6), (6, 2), (3, 4) or (4, 3). The result for this quadratic equation is 10 occurrences.

If the quadratic equation is:  $x^2 + 8x + c$  rational factors will only occur when  $c = 7, 12, 15$  or 16, however a product of 7 cannot be obtained when two regular dice are rolled. Checking the dice product lattice diagram a product of 12, 15 or 16 can be produced by any of the following dice combinations: (2, 6), (6, 2), (3, 4), (4, 3), (3, 5), (5, 3) or (4, 4). The result for this quadratic equation is 7 occurrences.

If the quadratic equation is:  $x^2 + 9x + c$  rational factors will only occur when  $c = 8, 14, 18$  or 20, however a product of 14 cannot be obtained when two regular dice are rolled. Checking the dice product lattice diagram a product of 8, 18 or 20 can be produced by any of the following dice combinations: (2, 4), (4, 2), (3, 6), (6, 3), (4, 5) or (5, 4). The result for this quadratic equation is 6 occurrences.

If the quadratic equation is:  $x^2 + 10x + c$  rational factors will only occur when  $c = 9, 16, 21, 24$  or 25, however a product of 21 cannot be obtained when two regular dice rolled. Checking the dice product lattice diagram a product of 9, 16 or 24 can be produced by any of the following dice combinations: (3, 3), (4, 4), (4, 6), (6, 4) or (5, 5) The result for this quadratic equation is 5 occurrences.

If the quadratic equation is:  $x^2 + 11x + c$  rational factors will only occur when  $c = 10, 18, 24, 28$ , or 30, however a product of 28 cannot be obtained when two regular dice are rolled. Checking the dice product lattice diagram a product of 10, 18, 24 or 30 can be produced by any of the following dice combinations: (2, 5), (5, 2), (3, 6), (6, 3), (6, 4), (4, 6), (5, 6) or (6, 5). The result for this quadratic equation is 8 occurrences.

If the quadratic equation is:  $x^2 + 12x + c$  rational factors will only occur when  $c = 11, 20, 27, 32, 35$  or 36, however products of 11, 27, 32 and 35 cannot be obtained when two regular dice are rolled. Checking the dice product lattice diagram a product of 20 and 36 can be produced by any of the following dice combinations: (4, 5), (5, 4) or (6, 6). The result for this quadratic equation is 3 occurrences.

Dice Sum & Factor Freq.	1	2	3	4	5	6
1	2 1	3 2	4 5	5 7	6 5	7 10
2	3 2	4 5	5 7	6 5	7 10	8 7
3	4 5	5 7	6 5	7 10	8 7	9 6
4	5 7	6 5	7 10	8 7	9 6	10 5
5	6 5	7 10	8 7	9 6	10 5	11 8
6	7 10	8 7	9 6	10 5	11 8	12 3

9. From the completed lattice diagram determine the theoretical probability for the number of quadratics that can be factorised in this dice simulation and compare this result to the estimated probability using the simulations.

There are a total of 226 equations that factorise successfully.

$$(1 + 2 \times 2 + 3 \times 5 + 4 \times 7 + 5 \times 5 + 6 \times 10 + 5 \times 7 + 4 \times 6 + 3 \times 5 + 2 \times 8 + 3 = 226)$$

There are 226 favourable outcomes from a total of 1296 possible. Probability of factorising a quadratic equation produced this way would be 226/ 1296 or approximately 17.4%.

**Extension**

What is the probability that a quadratic produced in the same way will factorise over the real number field?

If students understand the significance of the 'perfect square' they can identify equations of the form:

$y = x^2 + 8x + c$  can be factorised provided  $c \leq 16$ . In the dice product lattice diagram there are 26 possibilities.

Equations of the form:  $y = x^2 + 10x + c$  can be factorised provided  $c \leq 25$ . In the dice product lattice diagram there are 33 such possibilities.

Dice Sum & Factor Freq.	1	2	3	4	5	6
1	2 1	3 3	4 8	5 14	6 17	7 23
2	3 3	4 8	5 14	6 17	7 23	8 26
3	4 8	5 14	6 17	7 23	8 26	9 30
4	5 14	6 17	7 23	8 26	9 30	10 33
5	6 17	7 23	8 26	9 30	10 33	11 35
6	7 23	8 26	9 30	10 33	11 35	12 36

There are 765 favourable outcomes from a total of 1296 possible. Probability of factorising a quadratic equation produced this way would be  $765/1296 = 85/144$  or approximately 59%.

**Teacher Note:** A delightfully simple addition to this problem is the probability that a random quadratic generated in this way will factorise over the complex number system, no calculations required, the result is simply equal to a probability of one.

A more challenging question is to find the probability that a randomly generated quadratic (still with an  $x^2$  coefficient of 1) will factorise where no restrictions are placed on how the quadratic is generated. Would the answer be any different if an  $x^2$  coefficient other than 1 was considered?

An optional extension to this activity is to further investigate the notion of ‘sampling distributions’. Students generated 100 random quadratic equations and counted the ones that had rational factors. Problem 2 in the TI-Nspire file “Probable Factors” contains a program titled: ratsim(). The program can be called up by pressing the VAR key.

Run the ratsim() program and enter the sample size (number of quadratics to be generated) and the quantity of samples to be generated.

From a student’s perspective, the sample size relates to their by-hand exploration where they counted the number of quadratics that had rational factors.

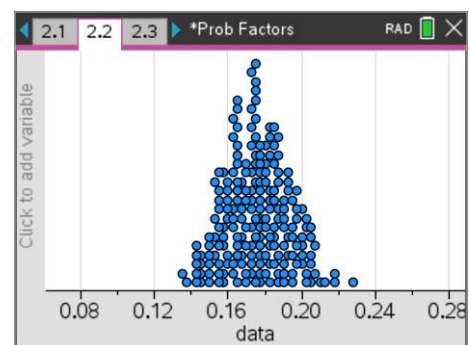
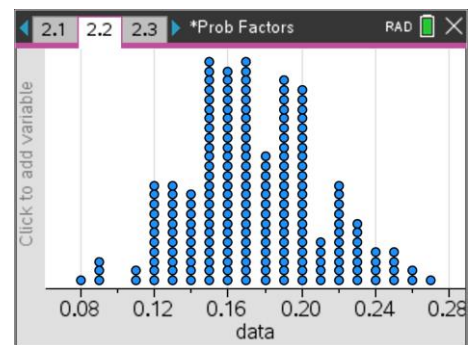
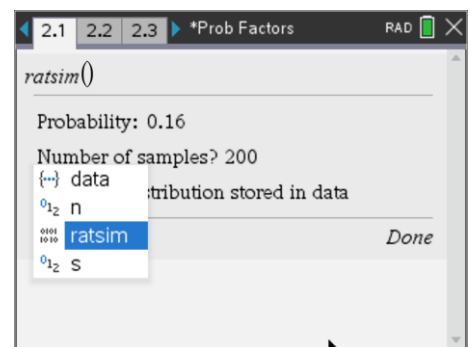
The quantity of samples corresponds to ‘how many students in the class’, equivalent to students sharing and comparing their results. Providing each student with a ‘sticky dot’ and placing it on the board as a scatterplot helps students understand what the calculator is simulating.

As the program runs it momentarily displays the proportion of quadratics with rational factors in each sample. These proportions are ‘readable’ on the handheld, but the computer software is generally too fast to read the values.

At the end of the program the proportions are stored in a list called: “data”.

The sample scatterplot shown opposite is for sample sizes of 100 (as per investigation) and the results of 200 samples.

Shown opposite is for sample sizes of 400 and the results of 200 samples. Notice how the distribution of the sample proportions is not as spread out. The sample proportions are ‘more predictable’ or deviate less from the mean.



The dotplot can be changed to a histogram. Use the analyse menu to superimpose a Normal Distribution curve. The calculator also displays the mean and standard deviation.

Notice how close the mean of the sampling distribution is to the theoretical mean of 0.174.

The sample shown opposite is for 400 samples each consisting of 100 quadratics.

This distribution provides an indication to students about the variability within their results and an excellent opportunity to discuss the 'likelihood' of their sample proportion being more than two standard deviations from the mean.

On page 3.1 of the Prob Factors document another program titled: `realsim()` simulates random quadratics with real factors.

The graph shown opposite is the sampling distribution for the proportion of quadratics with 'real factors'. (Sample size = 100)

Once again, notice how close the mean of the sampling distribution is to the theoretical value.

