

Activity Overview

In this activity, students will learn to quickly add lists of numbers by adding like terms and using the distributive property.

Topic: Numbers

- Distributive Property
- Greatest Common Factor
- Making conjectures

Teacher Preparation and Notes

- Prior knowledge of working with lists is helpful but not necessary.
- To download the student worksheet, go to education.ti.com/exchange/mgn

NORMAL FLOAT AUTO REAL RADIAN MP					
L1	L2	L3	L4	L5	2
1	7	-----	-----	-----	
2	5				
3					
4					
5					
6					
7					
8					
9					
10					
L2(3)=					

This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- The_Magical_Growth_of_Numbers_Student.pdf
- The_Magical_Growth_of_Numbers_Student.doc

Tech Tips:

- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Part 1 – Magic Sum Part 1

In this problem, students will discover the rule for the “*summagic*” problem by adding like terms and using the distributive property. They will use factoring to uncover the magic and explain the mathematics.

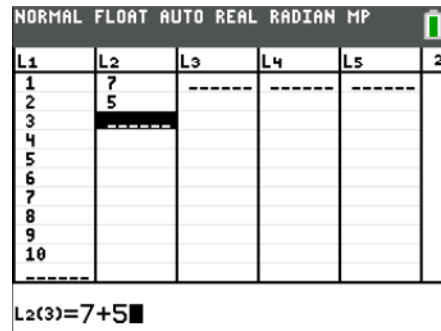
Questions 1–3

To complete the table in Question 1, have one student randomly select a number (between 1 and 10) for the first element. Have another student select a number for the second element. Then, each number after that is determined by adding the two previous numbers.

First, they will need to first enter the lists into **L1** and **L2**.

Press **[STAT]** and select **1:Edit...** to access the lists. If there are currently items in L1 and L2, move the cursor to the top of each list and press **[CLEAR]** **[ENTER]**. Then, enter the elements into the lists pressing **[ENTER]** between each number.

Once the random numbers are selected for elements 1 and 2 in L2, students can use addition to find the subsequent numbers. In this example, 7 and 5 are used. Element 3 can be found by adding 7 and 5 in the third position.



L1	L2	L3	L4	L5	2
1	7	-----	-----	-----	
2	5	-----	-----	-----	
3		-----	-----	-----	
4		-----	-----	-----	
5		-----	-----	-----	
6		-----	-----	-----	
7		-----	-----	-----	
8		-----	-----	-----	
9		-----	-----	-----	
10		-----	-----	-----	

L2(3)=7+5

Right after entering all the numbers, *magically*, tell the students the sum by mentally multiplying the 7th number by 11. Here’s a brief reminder of the multiply by 11 trick.

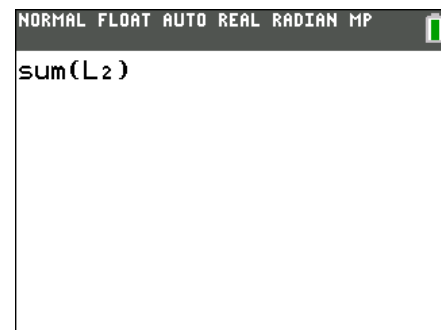
$$32 \times 11 = \begin{array}{r} 3 \quad 2 \\ \widehat{3} \quad \widehat{5} \quad 2 \end{array}$$

$$109 \times 11 = \begin{array}{r} 1 \quad 0 \quad 9 \\ \widehat{1} \quad \widehat{1} \quad \widehat{9} \quad 9 \end{array}$$

$$67 \times 11 = \begin{array}{r} 6 \quad 7 \\ \widehat{6} \quad \widehat{13} \quad 7 \\ 7 \quad 3 \quad 7 \end{array}$$

Then, have students find the sum of the list to verify your answer. On the Home screen, press **[2nd]** **[LIST]** **[>>]** and select **5:sum(**. Press **[2nd]** **[L2]** **[)]** **[ENTER]** to select **L2** and execute the command and show that the sum is equal to the sum you quickly gave.

Ask, “How was I able to find the sum so quickly?” Give students a chance to explore L2 and make conjectures. “Do you think I could do it again?” Have two other students choose two new numbers and repeat the process.



sum(L2)

Part 2 – Magic Sum Part 2

Question 4

After you have stunned your class by correctly finding the sum, develop the following on the table in Question 4 to investigate the teacher’s magic. Have the students complete the table along with you. Use the two numbers 7 and 5 to numerically find the next eight terms. Ask, “What pattern can you see? Can we write these in a briefer form?” Help the students find the next two columns.

Questions 5–12

After the table is completed, have the students answer the questions following the table. Before working on Question 12, make sure that all students fully understand the *summagic* rule (multiplying the 7th element by 11).

Extension

Develop the “magic” rule using variables and generalize the rule.

1	a	$= a$
2	b	$= b$
3	$b + a$	$= b + a$
4	$(b + a) + b$	$= a + 2b$
5	$(b + a + b) + (b + a)$	$= 2a + 3b$
6	$(b + a + b + b + a) + (b + a + b)$	$= 3a + 5b$
7

How can we find the sum of list two in terms of a and b ? How many a ’s are there? How many b ’s are there? Find the sum of all ten elements in **L2** by combining the number of a ’s and number of b ’s to get the sum. ($55a + 88b$)

What do you notice about the numbers in front of the variables? Lead the discussion to find that the numbers 55 and 88 are multiples of 11. Ask: How could we write this expression in an equivalent form? Using the distributive property the expression can be rewritten as:

$$55a + 88b = 11(5a) + 11(8b) = 11(5a + 8b)$$

- Do you notice the expression $5a + 8b$ in the list above? (It is the 7th element in **L2**.)
- How does the sum of the list compare to this term? (It is eleven times the term.)

Therefore, if you find the 7th element and multiply it by eleven you will have the sum of the list. Will this work for other numbers?

Have the students work in pairs to choose their own two numbers and repeat the activity. Ask them to find the sum using the *summagic* rule. Have them go to the Home screen and check their sum using **sum(L2)**. Check with each of the pairs to see if they were able to find the correct sum.

To wrap up, have a few of the groups explain to the class how they found their sum. Ask students to write the rule in their own words.



Solutions – Student Worksheet

Part 1

1–3. **Answer: Answers will vary based on numbers selected by students.**

Part 2

4. Complete the table based on the rules discussed, using the numbers 7 and 5.

Answer:

L1	Each Element	Distributive Property	L2
1	7	= 7	7
2	5	= 5	5
3	7 + 5	= 7 + 5	12
4	(7 + 5) + 5	= 7 + 2(5)	17
5	(7 + 5 + 5) + (7 + 5)	= 2(7) + 3(5)	29
6	(7 + 5 + 5 + 7 + 5) + (7 + 5 + 5)	= 3(7) + 5(5)	46
7	etc. ...	= 5(7) + 8(5)	75
8		= 8(7) + 13(5)	121
9		= 13(7) + 21(5)	196
10		= 21(7) + 34(5)	317

- In the table, how many 7s are there? **Answer: 55**
- How many 5s are there? **Answer: 88**
- Write the mathematical expression as: **Answer: $55 \times 7 + 88 \times 5$**
- What factor do 55 and 88 have in common? **Answer: 11**
- Rewrite the expression as: $(11)(5)(7) + (11)(8)(5) = 11(5 \times 7 + 8 \times 5)$. Verify that the expression is the same the one in Question 7. (Use your calculator to find the value of all three mathematical expressions.) Are the expressions the same? **Answer: yes**
- Is the number $5(7) + 8(5)$ in the numerically generated list in the table for Question 4? If so, what number in the list? **Answer: 7th**
- Use **sum(L2)** to find the sum of the numbers generated in your table. Press $\boxed{2nd}$ $\boxed{[LIST]}$ \blacktriangleright \blacktriangleright and select **5:sum(**. Press $\boxed{2nd}$ $\boxed{[L2]}$ $\boxed{)}$ \boxed{ENTER} to select **L2** and execute the command.
Sum = **Answer: 825**
What is the summagicrule? **Answer: multiply the 7th element by 11**
- Work with a partner to choose your own numbers and find the sum using the *summagic* rule. Check your sum using **sum(L2)**.

Answer: See students' work. Answers will depend on tables generated by students.