



## Estimated probabilities from simulating random sampling

On Page 1.5, use the 'draw' slider to simulate drawing 100 samples, with and without replacement. Use the results displayed in the dot-plot and in the histogram to complete the following tables in Questions 1 and 2.

### Question: 1

Complete Table 1 for the estimated probability that a sample, drawn **with** replacement, will contain 0, 1, 2 or 3 blue marbles.

**Table 1: With replacement**

| Quantity of blue marbles | 0 | 1 | 2 | 3 |
|--------------------------|---|---|---|---|
| Estimated probability    |   |   |   |   |

### Question: 2

Complete Table 2 for the estimated probability that a sample, drawn **without** replacement, will contain 0, 1, 2 or 3 blue marbles.

**Table 2: Without replacement**

| Quantity of blue marbles | 0 | 1 | 2 | 3 |
|--------------------------|---|---|---|---|
| Estimated probability    |   |   |   |   |

### Question: 3

What are some key similarities and differences for the data obtained 'with replacement' and 'without replacement'? Consider features such as the shapes and heights of the data displays, and the estimated probabilities displayed in Tables 1 and 2.

### Question: 4

Discuss why selecting the samples with replacement and without replacement lead to these differences, including in the estimated probabilities displayed in Tables 1 and 2.

## Variability of results for repeated simulations

On Page 1.5, click the 'reset' slider to clear the data. Simulate drawing a new set of 100 trials. Actual results are stored in the spreadsheet on page 1.7

|   | J_rep | B  | blu_nrep | C | blues | D         | prob...  | E        | prob_... |
|---|-------|----|----------|---|-------|-----------|----------|----------|----------|
|   | =     |    |          |   |       |           | prob_rep |          |          |
| 1 |       | 2. | 3.       |   | 0     |           | 0.       | 0.166667 |          |
| 2 |       | 2. | 2.       |   | 1     | 0.1666... |          | 0.166667 |          |
| 3 |       | 2. | 2.       |   | 2     | 0.6666... |          | 0.333333 |          |
| 4 |       | 1. | 1.       |   | 3     | 0.1666... |          | 0.333333 |          |
| 5 |       | 2. | 3.       |   |       |           |          | 1.       |          |

### Question: 5

Compare the data obtained from the new simulations with the results of the previous simulations. What similarities and differences are there?

The simulation results are summarised as lists on Page 1.7. The orange column shows the estimated probabilities of 0, 1, 2 or 3 blue marbles for sampling with replacement; the yellow column shows this information for sampling without replacement.

**Question: 6**

For the new set of 100 trials, compare the estimated probabilities on your Page 1.7 with that obtained by other students. Suggest ways in which the variability of results between successive simulations could be reduced.



**Increasing the number of trials in the simulation**

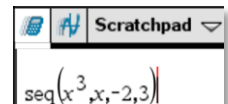
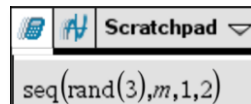
On Page 2.1, the list 'urn' is defined as {b,b,b,b,b,g,g,r,r}, and the commands 'randSamp(urn,3,0)' and 'randSamp(urn,3,0)' are contained within the nested instruction of the form 'countlf(randSamp(urn,3,x),b)'.

**Question: 7**

Explain the purpose of the commands:

- 'randSamp(urn,3,0)' and 'randSamp(urn,3,1)'
- 'countlf(randSamp(urn,3,0),b)' and 'countlf(randSamp(urn,3,1),b)'.

Open the Scratchpad of your TI-Nspire by pressing the  key. Explore the 'seq' command by inputting and evaluating the following. Press the  key to return to the main document.



**Question: 8**

Page 2.1 contains the commands 'seq(countlf(randSamp(urn,3,0),b),k,1,1000)' and 'seq(countlf(randSamp(urn,3,1),b),k,1,1000)'. Explain the purpose of these commands.

Each increment of the sliders labelled 'simulate' on Pages 2.1, 2.2 and 2.4 result in a new simulation of 1000 trials, for sampling with replacement and without replacement.

**Question: 9**

Observe the histograms on Page 2.2 as the 'simulate' slider is used for repeated simulations.

Comment on the variability between repeated simulations, as indicated by changes in the relative heights of the columns in the histograms.

**Question: 10**

From the heights of the columns of the histograms, estimate the probability of 0, 1, 2 or 3 blue marbles in samples selected with replacement and without replacement. From the table on Page 2.3, check your answer.

## Comparing theoretical probabilities with the simulation results

### Question: 11

Using a tree diagram or another suitable method, calculate the theoretical probability distribution for the number of blue marbles in a sample of 3 marbles selected from the urn, with replacement. Show these probabilities in the second row of Table 3.

Table 3: With replacement

| Quantity of blue marbles | 0 | 1 | 2 | 3 |
|--------------------------|---|---|---|---|
| Theoretical probability  |   |   |   |   |
| Estimated probability*   |   |   |   |   |
| Percentage error         |   |   |   |   |

\* Estimated from 10 000 trials

### Question: 12

Using a tree diagram or another suitable method, calculate the theoretical probability distribution for the number of blue marbles in a sample of 3 marbles selected from the urn, without replacement. Show these probabilities in the second row of Table 4.

Table 4: Without replacement

| Quantity of blue marbles | 0 | 1 | 2 | 3 |
|--------------------------|---|---|---|---|
| Theoretical probability  |   |   |   |   |
| Estimated probability*   |   |   |   |   |
| Percentage error         |   |   |   |   |

\* Estimated from 10 000 trials

Use the 'simulate' slider to obtain the results of repeated simulations, and observe these results on Page 2.3. These results are captured by a program on Page 2.5, and the mean of the repeated simulation results is displayed in the table on Page 2.4.

On Page 2.4, click the 'refresh' slider twice, then use the 'simulate' slider to accumulate the results of 10 repeated simulations, of 1000 trials each, for sampling with replacement and without replacement.

### Question: 13

In the third rows of Tables 3 and 4, record the estimated probabilities from Page 2.4, correct to four decimal places, based on 10,000 trials. Calculate the magnitude of the percentage errors in the estimated probabilities, correct to two decimal places, and record these percentage errors in the fourth rows of Tables 3 and 4.