Monopoly – Sum Dice

Teacher Notes & Answers

7 8 9 10 **11** 12

TI-30XPlus MathPrint™ Activity Student 45 min

Introduction

Teacher Notes:

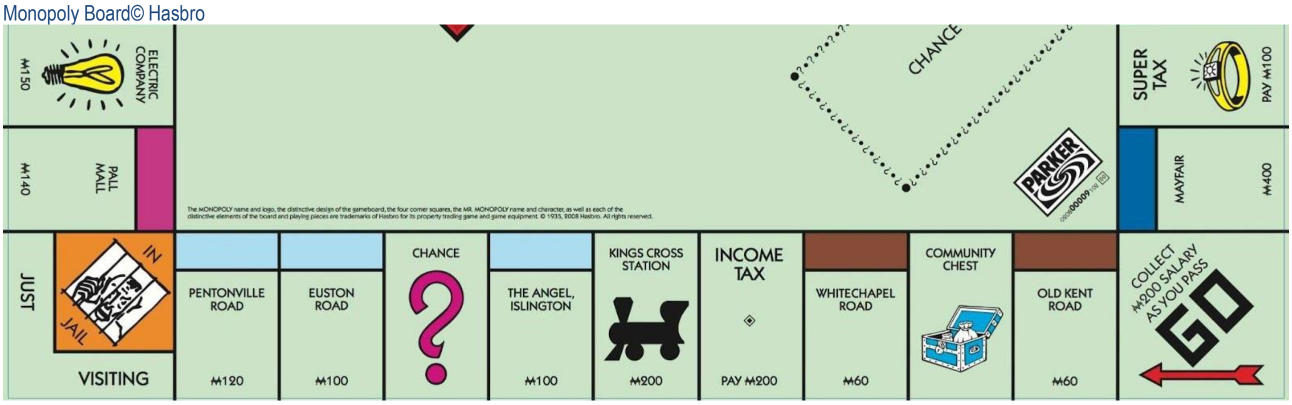
“Driving conceptual change regarding probabilistic reasoning through simulations is best achieved when students are encouraged to make predictions and then use their emprical results to compare, confront and reflect on their predictions.” [Tarr & Lannin, 2005]. Simulations are used to help develop critical concepts such as randomness, variation, central tendency, distribution, and the law of large numbers.

Teaching students how to use the calculator to generate simulations provides them with the skills to construct their own simulations. Students provided with this type of instruction are “capable of understanding the underlying assumptions of a probability simulation mainly with regard to evaluating the validity of a probability generator, constructing a valid probability generator to simulate various contextual, probabilistic situations and use the results of the simulation to calculate empirical probabilities.” [Zimmerman 2002]

This activity is built with this research in mind and therefore encourages students to make predictions, use the calculator to create simulations, reflect upon the results and compare the outcomes to emprical probabilities through the context of the board game Monopoly™.

In the game of Monopoly™ players seek to buy properties and charge other players rent whenever they land on their property. Rent is automatically increased if a player owns a set of properties (set = all the same colour). Rent can be further increased on a set if the player purchases houses or hotels. The overall aim is to develop a financial monopoly, essentially bankrupting other players. Players progress clockwise around the board (right to left in the image below) by rolling two dice and moving forward an amount equal to the sum.

Monopoly Board© Hasbro



Question: 1.

The following questions assumes play is just starting and all players are positioned on “GO”.

Note: “First Roll” refers to a single roll of the two dice and does not account for the free turn provided to a player that rolls a double.

i) What is the maximum number of squares a player might progress on their first roll?
Answer: 6 + 6 = 12 squares.

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ii) Which square(s) are not possible to land upon for the first roll?
Answer: Any square that is more than 12 spaces from GO. The first square is also impossible (Old Kent Road) as the minimum sum is 1 + 1 = 2.

iii) Which square do you think players are most likely to land upon for their first turn?
Answer: Answers may vary as the question asks for the students to 'guess'. Answers should however be 'reasonable'.

iv) How likely do you think it is that a player will land on one of the light blue set: The Angel Islington, Euston Road & Pentonville Road, in their first roll?

0	0% < B < 20%	20% < B < 40%	40% < B < 60%	60% < B < 80%	80% < B < 100%	100%
Impossible	Unlikely	Somewhat Unlikely	Approximately Even	Somewhat Likely	Likely	Certain

Answer: ($7/18 \approx 0.39$) Answers will vary, the question asks students to 'guess'. Students should not reason 'somewhat likely to certain'; there are only three squares (6, 8 & 9), they do not include the most likely (7) nor do they include square 5 (same probability square 9) or any of the other possible squares.

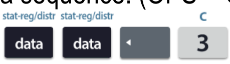
v) On average, how many squares do you think a player is likely to move forward on any given roll?
Answer: Answers will vary as the question asks for the student's to 'guess' or estimate, however '7' squares is the middle value and students should be able to reason that sums of 2 and 12 are unlikely and that the probabilities increase as you get closer to 7, furthermore the results will be symmetrical about this quantity.

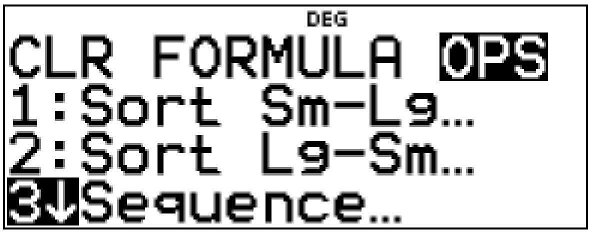
Generating Data

The first task is to simulate a single dice roll, generating whole numbers between 1 and 6. (inclusive).

Press  repeatedly to see the different numbers.

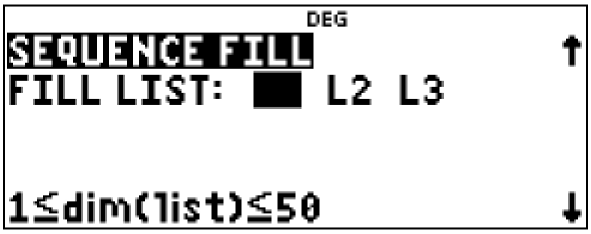
The calculator lists can be used to generate up to 50 samples at a time. The easiest way to do this is to generate a sequence: (OPS = Operations menu)





Select L1 (List 1) by using the navigation pad.

Press  to select and progress to the expression.



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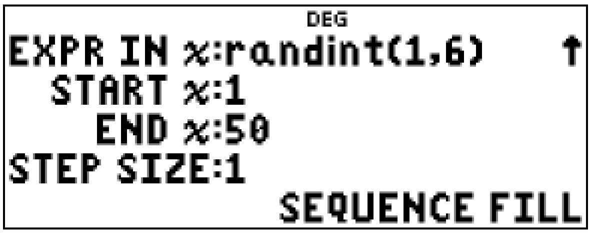
Use the **random** menu again to access the random integer command.

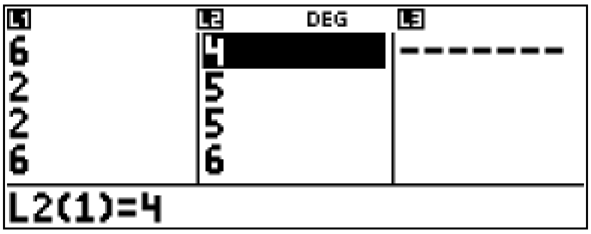
Start the sequence when x = 1 and end at 50.

Select **sequence fill** and List 1 will be populated with 50 random numbers between 1 and 6.



Repeat the above process to place 50 dice rolls in List 2.

In the sample shown opposite the sum of the first pair of dice rolls is 10, the second pair is 7, then 7 again and the fourth roll would be 12.

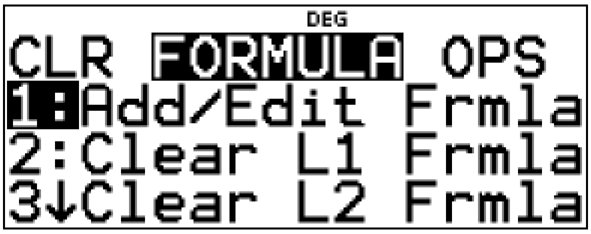


The dice rolls can be added automatically by using a list formula.

Press: stat-reg/distr data

Navigate across to List 3 then press: stat-reg/distr data

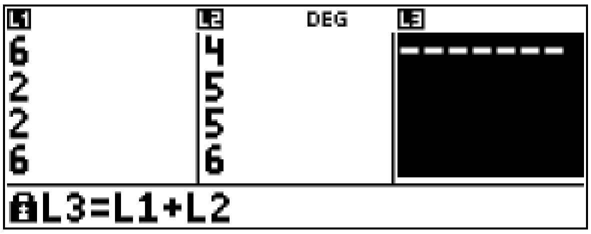
Navigate across to **FORMULA** select: **1: Add/Edit**.



The aim here is to add List 1 and List 2 together.

To access list names, press the stat-reg/distr data key.

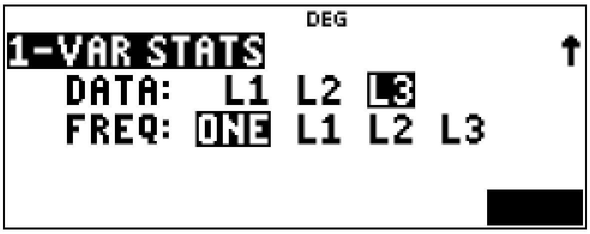
Once the data has been generated, you can scroll through the list of results (L3). To return to the calculator home screen, press: 2nd quit mode



Summary information useful for the following questions can be obtained using statistical analysis.

From the home screen, press: 2nd stat-reg/distr data B 2

Select List 3 and "ONE" for the frequency and then calculate.



Question: 2.

The following questions relate to the data in list 3, the sum of two dice:

i) What is the minimum sum rolled in your dice simulation? Discuss.

Answer: Answers will vary as this is a simulation, most students (≈75%) will have a minimum of 2. Students should discuss any differences between their answer to Q1.(ii) and the simulated result.

Teacher Notes: The discussion should include a statement on how the simulation sum was obtained (look through the data). Students should be encouraged to check results with friends, effectively increasing the sample size (simulation) and therefore reducing the likelihood that nobody had a minimum of 2.

ii) What is the maximum sum rolled in your dice simulation? Discuss.

Answer: Answers will vary as this is a simulation, most students (≈75%) will have a maximum of 12. Students should discuss any differences between their answer to Q1.(ii) and the simulated result.

Teacher Notes: The purpose of these two questions is to encourage students to think about the sample space, but still without any formal construction of a lattice or tree diagram to identify all possible outcomes and their respective probabilities.

iii) Based on your simulation data, how likely is it that a player will land on one of the light blue set in the first roll?

Answer: Answers will vary, however as the theoretical probability is 7/18 ≈ 39%, students should choose 'somewhat unlikely or approximately even', based on the scale provided in Q1.

Teacher Notes: Students should use their empirical data to *estimate* the probability.

iv) Based on your simulation data, what is the average number of squares a player is likely to move forward on any given roll?

Answer: Answers will vary, however as the theoretical quantity is 7, students answers should be close to this quantity.

Teacher Notes: Students should use the statistical calculation for the expected value from their statistics results on the calculator.

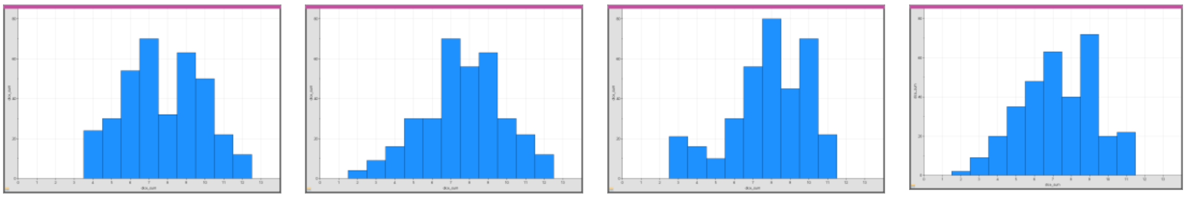
Question: 3.

Create a frequency table and histogram for your simulated data and comment on the results.

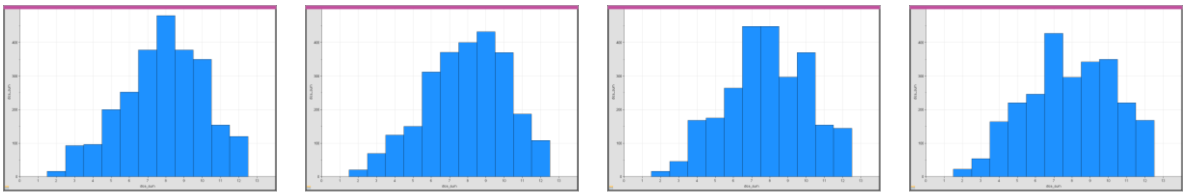
Answer: Answers will vary.

Teacher Notes: Student comments may include comments such as 'unlikely', but should not include comments such as 'bad data'. Sums such as 2 and 12 have only a 1/36 chance (each) of occurring, so it possible these will not appear in a data set. The mode will most likely be one of the central numbers: {5, 6, 7, 8 or 9} but not necessarily 7. It is quite likely the data will not be symmetrical given the relatively small sample size (50). The frequency table however **must** correspond to the histogram.

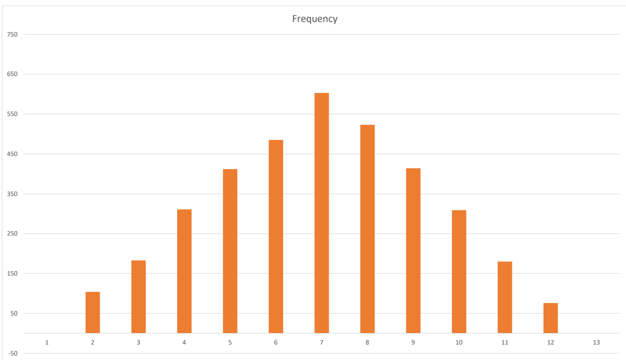
Sample histograms (50 data points) are shown below.



The samples shown below represent simulations for 360 dice rolls.



Much larger sample sizes are required to produce the symmetry associated with the long term theoretical results. A spreadsheet that generates 3600 sums is included in the file set for this activity. Pressing F9 on the spreadsheet generates a fresh simulation each time. (Sample shown below)



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Calculating Probabilities

Theoretical probabilities can be computed by populating a sample space and identifying all possible outcomes.

Sample Space - A sample space is a set of all possible outcomes of a random experiment.

Question: 4.

Copy and complete the table below for the sample space for the sum of two dice.

Answer:

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Question: 5.

Use your previous sample space to complete the probability distribution table and corresponding expected value for the dice sum.

x	2	3	4	5	6	7	8	9	10	11	12
$\mathrm{P}(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
$x.\mathrm{P}(x)$	2/36	6/36	12/36	20/36	30/36	42/36	40/36	36/36	30/36	22/36	12/36
$\sum x.\mathrm{P}(x)$	252/36 = 7										

Question: 6.

Use the sample space and probability distribution table to answer the following:

i) What are the minimum and maximum numbers that can be generated for the sum of two dice.

Answer: Minimum = 2, Maximum = 12

ii) What sum is most likely to be generated when two dice are rolled, and which square does this correspond to on the first side of the board?

Answer: Most common (likely) number = 7, Pr(X=7) = 1/6, this corresponds to the 'Chance' square.

iii) What is the probability that a player will land on one of the light blue properties on the first roll?

Answer: The player would need to roll either a 6, 8 or 9, 5/36 + 5/36 + 4/36 = 14/36 ≈ 39%.

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iv) What is the probability that a player will roll a double number?

Answer: There are 6 equally likely 'doubles': (1, 1); (2, 2); (3, 3); (4, 4); (5, 5) and (6, 6). The probability of a specific double number 1/36, therefore the probability of any double number is: 6/36 = 1/6.

v) On average, how many squares does a player move forward when they roll a double number?
Note: Do not include subsequent rolls, only consider the first roll.

Answer: There are 6 equally likely 'doubles': (1, 1); (2, 2); (3, 3); (4, 4); (5, 5) and (6, 6). The average number of squares to move forward given a double is rolled is therefore (2 + 4 + 6 + 8 + 10 + 12) ÷ 6 = 7

Question: 7.

In the game of Monopoly a player receives a second turn if they roll a double. If they roll another double (second double) then they receive another turn, however if they roll a third double, then they do not get to roll again or progress, indeed, they go straight to jail. Based on this information (ignoring forward/backward movement as a player heads to jail), what is the average number of squares a player will move forward each turn in the game of Monopoly.

Answer: $7+\frac{1}{6}\times 7+\frac{1}{36}\times 7=8\frac{13}{36}\approx 8.361$

Question: 8.

Daniel and Leah are trying to produce a simulation in List 1 only for the sum of two dice on their calculator.

Daniel's expression: randint(2,12).

Leah's expression: randint(1,6) + randint(1,6)

i) What numbers will be produced by Daniel's expression?

Answer: d = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

ii) What numbers will be produced by Leah's expression?

Answer: L = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

iii) Explain why one of the simulations is not correct.

Answer: Daniel's expression doesn't work as each of the numbers 2, 3 … 12 will be equally likely to be generated.

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