Math Objectives

- Students will use a moveable line to find a line of fit for a data set.
- Students will interpret the slope and *y*-intercept of the equation of their line of fit.
- Students will make predictions using their line of fit.
- Students will use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept (CCSS).
- Students will model with mathematics (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- slope
- *y*-intercept
- line of fit

About the Lesson

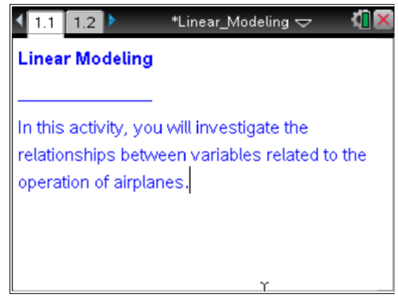
- This lesson involves modeling relationship between variables related to the operational cost of airplanes.
- As a result, students will:
  - Find the line of fit to model relationship between airplane operational cost and other variables.
  - Interpret slopes and *y*-intercepts in context of the data.
  - Make predictions based on lines of fit for the given data.

TI-Nspire™ Navigator™

- Send and collect documents.
- Quick Poll to assess student understanding.
- Class Capture to make comparisons of lines of fit and generate student discussions.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Linear_Modeling_Student.pdf
- Linear_Modeling_Student.doc

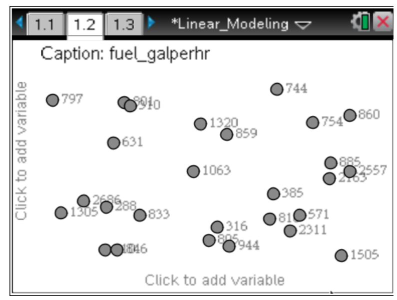
TI-Nspire document

- Linear_Modeling.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty rotating a line, check to make sure that they have moved the cursor until it becomes ⟲. If students have difficulty translating a line, check to make sure that they have moved the cursor until it becomes ✥. Then press ctrl to close the hand. Use the TouchPad to rotate or translate the line.

Move to page 1.2.



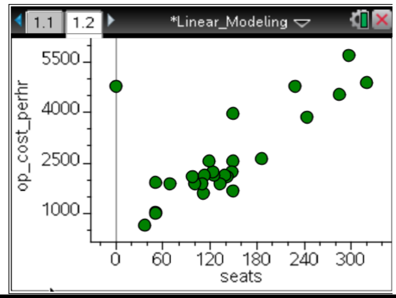
1. Construct a scatter plot for the number of passenger seats vs. the operational cost of an airplane:
   a. Select *add variable* at the bottom of the page where the horizontal axis should be placed, and select the variable **seats**.
   b. Select *add variable* on the left of the page where the vertical axis should be placed, and select the variable **op_cost_perhr**.

Teacher Tip: Variables are stored in the document as lists, so students can select them to be plotted on the coordinate axes. Initially, the axes are not shown on the Data & Statistics page. When students select the area at the bottom of the page where a horizontal axis should be placed, they will see the choice of stored independent variables. After they select the independent variable, they should move the cursor to the left of the page and select the area where a vertical axis would be placed and select operational cost as the dependent variable. The scatter plot with labeled coordinate axes will then appear. In order for students to see actual data, they can insert a Lists & Spreadsheet page and type the names of the variables in the headings of the columns.

Tech Tip: After the scatter plot is constructed, students can swipe over a data point to reveal its actual coordinates.

2. Explore the scatter plot and answer the following questions:

   a. Describe the relationship between the number of seats and the operational cost per hour displayed in your graph.



<u>Answer:</u> Positive relationship, fairly linear, one data point (0, 4804) is different from the apparent relationship.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

   b. Why might there be a relationship between the number of passenger seats and the operational cost of an airplane?

<u>Sample Answers</u>: An airplane designed to carry a larger number of passengers is larger and thus uses more fuel.

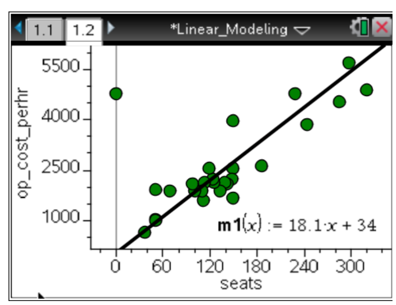
   c. Describe a situation for which the data point (0, 4804) would make sense.

<u>Sample Answers</u>: A cargo plane has no passenger seats, but it costs $4,804 per hour to operate it.

   d. Explain why the data point (0, 4804) should or should not be used when fitting a line to the data.

<u>Sample Answers</u>: This data point should not be used because the line is trying to establish a relationship between the number of passenger seats in the plane and the cost of operation. This data point is for a cargo plane, not a passenger plane.

3. To explore the relationship between the variables, add a movable line by following the handheld steps below:



   - Select **MENU > Analyze > Add Movable Line.** A line will appear on the screen.
   - Move your cursor until it is near what appears to be the end of the line. A will appear.
   - Press ctrl to grab the line and rotate it.
   - Press esc or to release the line.
   - Move your cursor until it is near what appears to be the middle of the line. A will appear.
   - Press ctrl to grab the line and move it horizontally and vertically.
   - Move the line until you think it best represents the data. Press esc.

Tech Tip: Select > **Analyze** > **Add Movable Line.** To move a line graph, Grab and hold either end of the line to rotate it. Grab and hold the middle of the line graph to move the entire line.

a. What is the equation of your line?

Sample Answers: $m1(x) = 18.1x + 34$

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

b. What do the slope and *y*-intercept of the equation of your line represent in the context of this situation?

Sample Answers: For the equation given in 3a, the slope is 18.1. It means that the operational cost per hour increases by $18.10 for each 1 seat added. The *y*-intercept is 34. It means that the airplane with 0 passenger seats has an operational cost of $34 per hour.

c. Estimate the operational cost per hour that might be used by an airplane carrying 200 passengers. Explain how you got your answer.

Sample Answers: For the equation given in 3a, it is about $3650 per hour for this example.

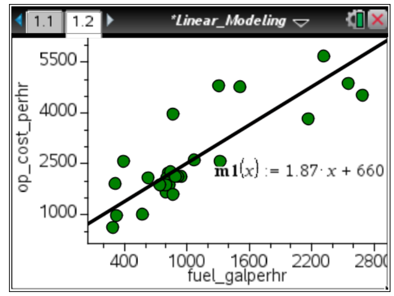
Tech Tip: Students can insert a Calculator page in order to complete calculations. Alternatively, students can use Scratchpad by pressing . In order to close Scratchpad and return to the document, select ☒ in the upper right corner. If students insert Calculator page, they can use the fact that the equation of the best fit line is stored as **m1**. To find the operational cost for the given number of seats they can just type **m1**(200) to get an answer. If students use Scratchpad, they do not have access to stored equations and have to type the complete expression in order to answer this question. Students could also estimate the operational cost from their graph.

Tech Tip: To complete calculations, students can add a Calculator page by selecting **+** at the top of the screen..Then select Calculator.

d. What other variables could affect the operational cost of an airplane?

Sample Answers: Luggage, speed, fuel mileage, flight length, etc.

4. How is the operational cost per hour affected by the amount of fuel used per hour? To change the variables graphed along the x-axis, select the bottom of the page under the horizontal axis and select the variable **fuel_galperhr**.



a. Adjust the movable line to fit the data. What is the equation of your line of fit?

Sample Answer: *m1(x)* = 1.87*x* +660

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

b. What is the *y*-intercept for your equation? What is the real-world meaning of the *y*-intercept of your graph?

Sample Answers: Answers will vary. For the equation given in question 4a, the *y*-intercept is (0, 660). This means that when 0 gallons of fuel are used per hour, the operating cost is $660 per hour — even when no fuel is burned in an hour, there are still costs associated with the airplane.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

c. What is the slope of your equation? What is the real-world meaning of the slope of your graph?

Sample Answers: Answers will vary. For the equation given in question 4a, the slope is 1.87. This means it costs $1.87 for every gallon of gas to operate the aircraft.

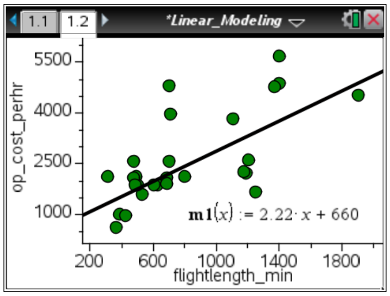
d. Using your line of fit, predict the number of gallons per hour used for a flight if the operating cost is $3,500 per hour.

Sample Answers: Answers will vary. For the equation given in question 4a, the amount of fuel used is approximately 1,519 gallons per hour.

Tech Tip: Students can substitute given values for *y* into the equation of the line and solve the equation for *x* in order to answer this question. Using a Calculator page, students can find the number of gallons per hour for a given operational cost by typing **solve(m1(x)=3500,x)**.

Teacher Tip: The question 3c asked students to find the output (dependent variable) given the input (dependent variable). This is equivalent to evaluating an expression given the value of the variable. Thus, all they needed to do was the direct substitution. The question 4d, however, asked students to find an output (dependent variable) given an input (independent variable). This is equivalent to solving a linear equation for unknown. Thus, students could substitute values for the output and solve the equation for the input, or algebraically change the equation to express input in terms of output and then substitute the given output to evaluate the new expression.

Teacher Tip: The relationship between the hourly operational cost and length of flight that students explore in this part of the activity is not very linear, as these are real-life data. The linear regression gives a linear equation of 2.19x +83 with the r value of .679 and r^2 is .461. Thus, the student answers might vary significantly. Even though the correlation between these two variables is weak, these variables have conceptually meaningful relationship. Students intuitively understand that the cost of flight operation should be increasing as the flight time increases, and thus modeling this relationship with a linear function is justified. In addition, students should have an opportunity to model real-life data and discuss reasons for linear modeling based on meaningful relationships and not only the appearance of the scatter plot of data.



5. And now, how is the operational cost per hour is affected by the flight length? To change the variables graphed along the x-axis, select the bottom of the page under the horizontal axis, and select the variable **flightlength_min**.
   a. Adjust the movable line to fit the data. What is the equation of your line of fit?

   <u>Sample Answers</u>: $m1(x) = 2.22x + 660$

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

b. What is the *y*-intercept for your equation? What is the real-world meaning of the *y*-intercept of your graph?

Sample Answers: Answers will vary. For the equation given in question 5a, the *y*-intercept is (0, 660). This means that when the plane did not take off the operating cost is $660 per hour — even without flying, there are still costs associated with the airplane.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of the document

c. What is the slope of your equation? What is the real-world meaning of the slope of your graph?

Sample Answers: Answers will vary. For the equation given in question 5a, the slope is 2.22. This means that hourly cost is $2.22 for every minute of flight to operate the aircraft.

> **Teacher Tip:** This question could be hard for students, since the slope represents the amount of change of the hourly operational cost with each minute of flight. The hourly operational cost is already a rate, and this relationship is really showing the change in the hourly rate with flight time. The actual units of the slope are $/hr per min or $/min^2. If students have difficulty with understanding the slope meaning in this situation, use an analogy of an accelerating car and ask how much speed is changing with time given rpm value. The analogy of cost of operation is distance, the analogy of hourly cost of operation is speed, and the analogy of the slope in the relationship above is the acceleration. Since acceleration means rate of change of speed with time, the slope in the relationship means rate of change of hourly cost with flight time.

d. Using your line of fit, predict the operational cost for a 2-hr flight.

Sample Answers: Answers will vary. For the equation given in question 5a, the cost is about $926 per hour.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to find a line of fit for a data set.
- How to interpret the slope and *y*-intercept of the equation of a line of fit in a real-world context.
- How to make predictions using a line of fit.

TI-Nspire Navigator

Note 1

Name of Feature: Quick Poll

Use an open-response question in Quick Poll to collect students' answers to questions for class discussion.

Note 2

Name of Feature: Class Capture

Use Class Capture to show student lines of fit. Discuss how they are alike or different.