Finance Problems

Introduction

Would you like to retire with a million dollars cash in your account? No need to win the lottery or meet with any shady characters. You can achieve this goal by saving just $60.00 per week! To understand how this is possible, you need to have an understanding of recursive relationships. We generally think in linear terms, interest on loans, annuities and investments however are not linear. Over extended periods of time a curve can deviate a long way from a straight line.

Simon would like to become a millionaire and therefore looks at the savings plan this way: (Linear)

> $**60.00** per week
> **52** weeks per year
> 65 – 17 = **48** years (65 = Simon's planned retirement age)
> 60 x 52 x 48 = $149,760.00
> Where is the extra $840,240.00 coming from?

The first thing Simon has missed is interest. Let's suppose Simon's account attracts an annual 5% flat rate of interest.
Simon rethinks his calculations:

> $60.00 per week
> $60 \times 52 = \$3,120.00$ (Annual savings)
> 60 x 52 $\times$ 0.05 = $156.00 (Interest on $3,120 for one year)
> ($3,120 + 156.00) $\times$ 48 = $157,248.00 (Savings 48 years)

There are several problems with Simon's second calculation. The first is that he is still trying to think in a linear way. Simon's calculation his annual interest is based on the having $3,120.00 invested for a whole year, but he only reaches this amount at the end of the year, in this case the actual interest would be smaller. Simon then reapplies his linear logic for the 48 years, this time he misses the 'compounding' interest. Simon is not thinking about the interest that he will earn on the progressively increasing balance each year. Interest calculated on an increasing balance and accumulating interest is referred to as compounding. Compounding interest is recursive, that is, each interest calculation is based on the closing balance at the end of the previous month producing a non-linear progression.
Simon's friend Colleen starts by explaining how compound interest works.

> **Week 1**
> $60.00 (weekly deposit)
> 5% interest rate per annum. (5% / 52 weeks)
> $60.00 x 5% / 52 = $0.06
>
> **Week 2**
> $60.06 (balance from previous week)
> 5% interest rate per annum. (5% / 52 weeks)
> $60.06 x 5% / 52 = $0.12

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The small quantities represented here and the short duration for the investment (2 weeks) do not allow us to see the difference, unless you include the additional decimal places. However, we can still see that interest in the second week is being calculated on the larger amount of money in the account. As this amount increases so too will the computed interest. Think of each week as the term in a sequence where the next term is dependent on the previous. For simple interest, the change each week is the same (common difference), for compound interest the change each week is different (non-linear). This small change doesn't seem like much initially, but spread out over 48 years, this amount can make a significant difference.

Simon is still struggling to see the big picture, there is a lot going on here. Colleen decides to break things down:
- Simple interest
- Compound interest
- Annuities[1] (Saving for the future)

Part A: Simple Interest
Let's say that Simon has invested $3,120 into a term deposit account that attracts a flat rate of 5% per annum, remember flat rate is the same as "simple interest rate". This means the original amount invested is the first value of the recursive relationship, v_0. The amount of interest is calculated by working out 5% of $3,120. This gives us a value for d, the common difference between consecutive account balances.
The recursive relationship would be:

$v_0 = 3120$,
$v_{n+1} = v_n + 156$ (5 % of $3120 = $156)

Open the TI-Nspire (tns) file provided: **Finance Problems**.

On page 1.1 and you will see that 3120 is entered as v_0.

The next page (Page 1.2) shows a spreadsheet with the calculations for a 10 year investment period.

The formula in cell B2 states: = b1 + d

The formula in cell B3 states: =b2 + d

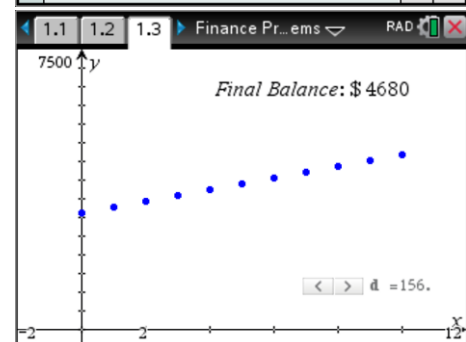Each successive term adds the common difference: 'd' ($156) to the previous term.

The next page (1.3) is a graph showing how the investment is growing over time. Notice that the points form a straight line, the growth of the investment is 'linear'.
The value of d can be changed by clicking on the arrow or by editing the value directly.
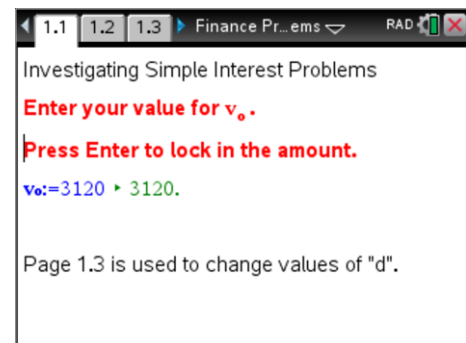
[1] Annuity – This activity focuses on building the investment rather how it reduces as the fund is drawn down.

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To see the computed balance at any stage press:

[menu] > Trace > Graph Trace

Use the left and right arrow keys on the keypad of the calculator to move from one point to another. From the screen shown opposite it can be seen that after four years the value of this investment was $3,744.
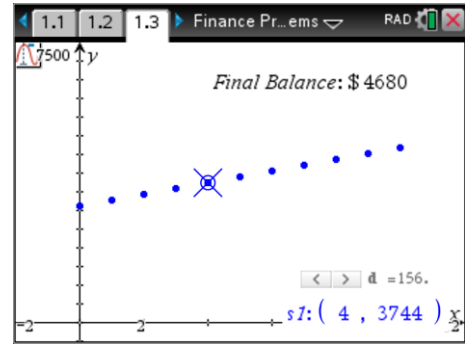
Teacher Notes:

The aim is for students to explore what happens to the investment if the interest rate is changed, specifically that for simple interest the result is linear. Changing the simple interest rate means that students need to use a different value for: d. The value of d can be changed using the slider on page 1.3. The initial value invested, v_o can be changed on page 1.1.

Note that for Questions 1 & 2 the sequence commands can be used, this is left up to the individual to determine how this is calculated. Students may also use the 'ans' (answer) functionality on the calculator application to simply add the common difference to 'ans' (previous answer) or they might edit v_o and d accordingly.

Question: 1

$8,000 has been invested into a term deposit that earns 3.5% simple interest annually.

a) Write down the recursive relationship that models this problem.

$v_0 = 8000$, $v_{n+1} = v_n + 280$

b) Use the calculator file to find the value of the investment after 6 years.

$v_6 = \$9680$

Question: 2

$6,000 has been invested into a term deposit that earns 5% simple interest annually.

a) Write down the recursive relationship that models this problem.

$v_0 = 6000$, $v_{n+1} = v_n + 300$

b) Use the calculator file to find the value of the investment after 10 years

$v_6 = \$9000$

Part B: Compound Interest

Suppose our original investment of $3120 attracts compound interest, compounding annually at the rate of 5%. The recursive relationship is shown below:

$v_0 = 5000$, $v_{n+1} = 1.05v_n$ $(r = 1 + 0.05)$

Problem 2 on the file allows us to investigate compound interest.

Page 2.1 on the file allows us to type in the initial investment of $3,120.

Page 2.2 shows the original simple interest and the eventual balance after 10 years. The dotted blue line shows the growth of the compound interest investment and the final (closing) balance.

You can change the interest rate and the number of compounding periods (cpy) to see how each one impacts the overall balance and 'curvature' of the financial growth.

Note: The value of r is calculated automatically as the interest rate and number of compounding periods per year (cpy) is changed.

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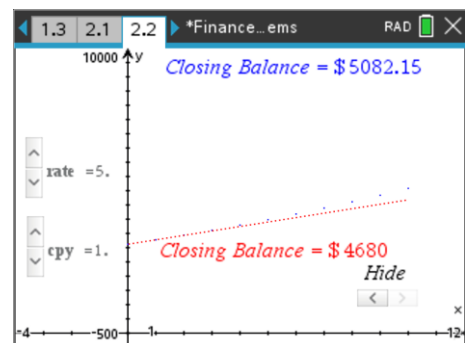
Question: 3

Simon invests $3,120 for a period of 10 years with a flat rate of interest of n%. Colleen invests the same amount of money, for the same amount of time but her interest is compounded annually at n%. After 10 years Colleen has a balance of more than $9,690.25.

    a) Use the 'rate' slider to determine the interest rate used by both Colleen and Simon.
       Using the slider, the interest rate will be 12%

    b) How much better off will Colleen be after 10 years?
       Colleen will have $9,690.25 and Simon will have $6864.00, so Colleen will be $2,826.25 better off.

    c) Colleen negotiates monthly compounding periods (12 per year) with the same interest rate. How much will she have at the end of 10 years for her original investment of $3,120?
       Using the slider, changing cpy to 12, the closing balance is $10,297.20.

    d) Explain, with reference to the graph, why the difference in their investments is not significant after one year, but very different after ten years.
       Simon's investment is linear. It goes up by the same amount each year. Colleen's investment is non-linear. The longer the investment remains in place the further the curve bends away from the straight line.

Compound Interest Tables

The graph on page 2.2 was created using a spreadsheet or compound interest table. In this section of the activity you will create your own compound interest table in a spreadsheet, learn about cell references and how to quickly generate a recursive list.

Teacher Notes:

    There are several ways this activity could be done in a Spreadsheet. Absolute cell referencing techniques are available in Excel and also the TI-nspire Lists and Spreadsheet application. Another technique, also available in Excel and TI-nspire is to reference a variable name, the latter technique is being used here.

Navigate to Page 3.1, a Notes application contains a list of the variables that need to be considered when computing interest on investments, specifically where compound interest is being calculated.

Enter the quantities shown opposite for the three variables:

- V_o = Opening amount of the investment

- Rate = Interest rate

- CPY = Number of compounding periods per year
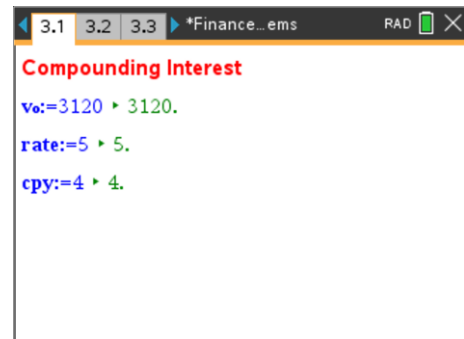
Navigate to page 3.2 where a spreadsheet has been set up to generate the compound interest table. Three columns have been created:

- **Op_bal** = Opening balance, amount of money at the start of each interest period.
- **Interest** = Amount of interest calculated for the specified time period. For example, if CPY = 4, then the interest would be for a 3 month period.
- **Cl_bal** = Closing balance, the amount of money in the account after interest has been added. This of course becomes the opening balance for the next time period.
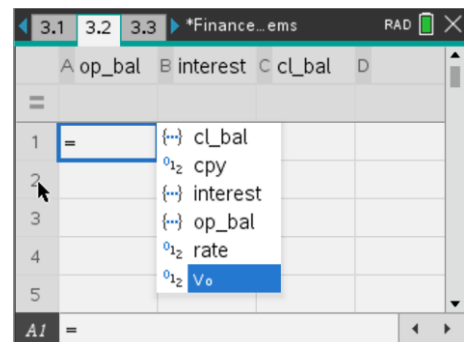
Cell A1 needs to contain the original deposit amount. All formulas must start with an equals sign (=), then select v_o by from the variable menu followed by [enter].

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The next step is to calculate the interest for the given time period.

As this represents another calculation it must start with an equals sign:

= a1 × rate ÷ (100 × cpy)

Note that the division by '100' is to convert the 'rate' (interest rate) variable into a percentage and the division by CPY converts the interest into the selected time period.

Once the interest has been calculated it can be added on to the balance or 'closing balance'.

Cell C1 needs the formula:

= a1 + b1

The closing balance (C1) needs to be transferred to cell A2 as the next 'opening balance'. Insert an appropriate formula in cell A2.

Once the formula is in cell A2, select the cell again and use the menu to generate corresponding formulas in cells A3 to A48.

The formulas in cells B1 and C1 also need to be extended down to cells B48 and C48 respectively. By holding down the shift key cells B1 and C1 can be highlighted, then Fill and arrow Down as before.

To check the balance of the investment you can call up an amount from the calculator application.

For example, using the information entered on Page 3.1, the balance for the investment of $3,120 for 10 years, compounding 4 times per year (quarterly) can be recalled by entering:

cl_bal [40]

The cl_bal (closing balance) variable can be recalled from the VAR menu. The '40' refers to 4 interest periods per year for 10 years.
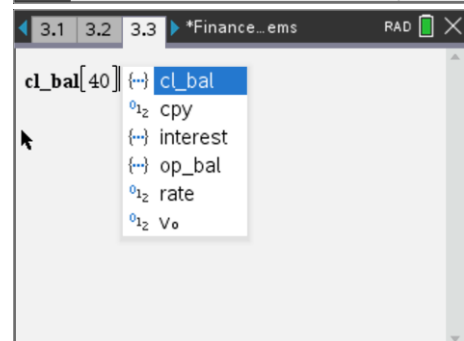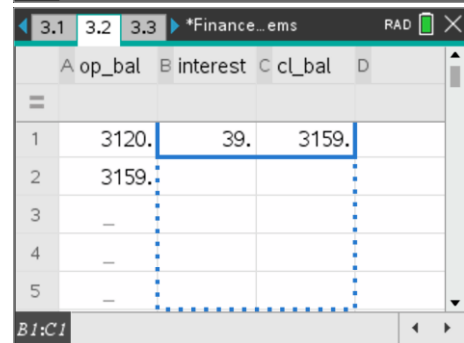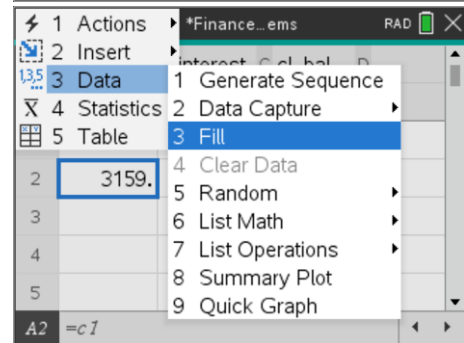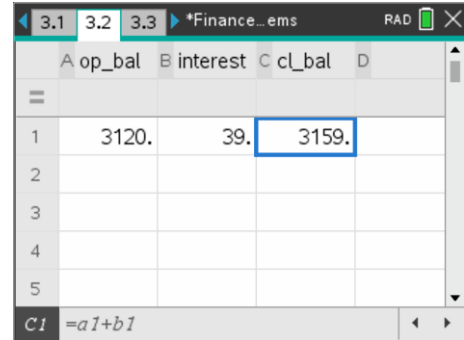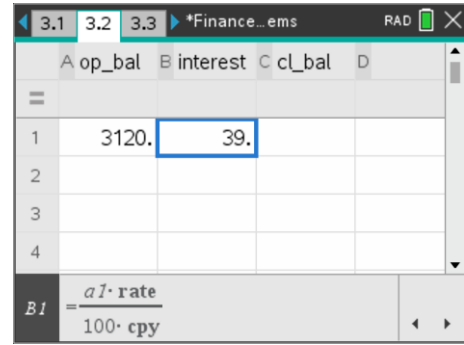
Question: 4

Simon invests $3,120 for a period of 10 years with a flat rate of interest of 10%. Colleen invests the same amount of money, for the same amount of time with the same interest rate with quarterly compounding periods.

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a) Explain why the amount of interest calculated in column B changes with each payment period.

As the opening balance increases (previous interest amounts added) the amount of interest increases as it is the same percentage but of this larger quantity.

b) How long does it take Colleen to reach the same Simons final balance?
Hint: Return to the spreadsheet and scroll down to find the closest corresponding balance.

Approximately 7 years. C28 = $6229.06 (Require to reach $6240.00)

c) In the calculator application enter the following expression:

$$3120 \times \left(1 + \frac{0.1}{4}\right)^{28}$$

What is this expression calculating?

This calculates the same as the spreadsheet. 28 = 7yrs × 4 CPY and 1 + 0.1/4 is the compounding interest.

d) Solve the following expression and explain the result:

$$solve\left(3120 \times \left(1 + \frac{0.1}{4}\right)^{n} = 6240, n\right)$$

What is this statement calculating?

N ≈ 28.07, the quantity of compounding interest periods required for Colleen to double her original investment of $3120 with an interest rate of 10% compounding quarterly.

e) Solve the following expression and explain the result:

$$solve\left(\left(1 + \frac{0.1}{4}\right)^{n} = 2, n\right)$$

What is this calculating?

This computes the amount of time it takes to double the original investment amount, which of course is the same as the previous computation. The amount of time it takes to double the original amount is dependent on the interest rate and the number of compounding periods per year.

Part C: Who wants to be a millionaire?

As the introduction to this activity stated, for an investment of just $60.00 per week it is possible to become a millionaire! To see how this is done the Spreadsheet on page 3.2 will be modified in order to include weekly contributions.

Navigate to page 3.1 (Notes), insert a Maths Box

[menu] > Insert > Maths Box

A new variable needs to be created to represent the payment contributions each week.

pmt := 60

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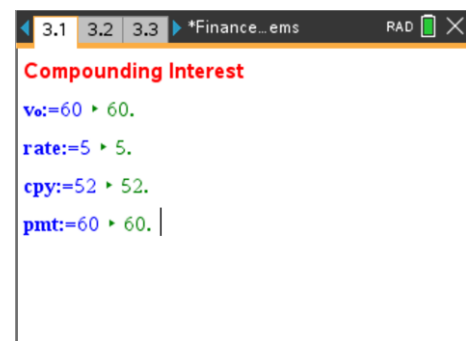
Navigate to page 3.2 (Spreadsheet) and edit the formula in cell C1 so that it includes the payment amount (pmt).

= a1 + b1 + pmt

Press [enter] to confirm the change to the cell formula, select the cell and then fill the formula down.

Note:

Extend all three columns to row 260.
This will provide calculations for 5 years (52 x 5 = 260).

	A op_bal	B interest	C cl_bal	D
=				
1	60.	0.05769...	'+b1+pmt	
2	120.057...	0.11544...	120.173...	
3	120.173...	0.11555...	120.288...	
4	120.288...	0.11566...	120.404...	
5	120.404...	0.11577...	120.520...	

C1 =a1+b1+pmt

Question: 5

Simon deposits his first $60.00 into his savings account and commences weekly deposits of $60.00. The bank has provided an interest rate of 7% for the account, compounded weekly aligning to his contributions.

a) How much money will Simon have in his account at the end of 5 years? (260 compounding periods)
According to the spreadsheet: C260 = $18,748.68

b) Given that Simon's investment strategy is for 48 years, explain why he has not even reached 10% of his target savings.
Compounding interest is not linear, so it doesn't go up by the same quantity for the same time period, therefore it is not possible to simply multiply the balance at 5 years to get the balance after 50 years.

Navigate to page 3.3 (Calculator) and use the Finance Solver to explore the long term investment. Use the [tab] key to navigate through the dialogue box and press [enter] on the item to be calculated.

Finance Solver

N:	48*52
I(%):	7.
PV:	60.
Pmt:	60.
FV:	-1000000.
PpY:	52

Edit Number of Payments, N

N = 48 × 52 (Total **N**umber of payments)

I = 7 (**I**nterest rate)

PV = $60 (**P**rincipal **V**alue *deposited* into the account)

PMT = $60 (Weekly **p**ay**m**en**t** *deposited* into the account)

FV = $ -1,000,000 (**F**uture **V**alue, the amount *returned* from the account)

PpY = 52 (**P**ayments **p**er **Y**ear, weekly = 52)

CpY = 52 (**C**ompounding **p**er **Y**ear, compounded weekly)

Question: 6

Simon deposits his first $60.00 into his savings account and commences weekly deposits of $60.00. The bank has provided an interest rate of 7% for the account, compounded weekly aligning to his contributions.

a) Use the Finance Solver to determine how much money Simon will have in his account at the end of 5 years and compare it to the amount determined by the spreadsheet. (260 compounding periods)
According to the Finance Solver: $18,748.68 (Same quantity as spreadsheet)

b) Use the Finance Solver to determine how much money Simon will have after 48 years.
N = 2496, I = 7%, PV = $60, PMT = %60, PpY = 52, CpY = 52 ... FV = $1,237,431 (1.2 million!)

Extension

Suppose Simon wants to use his million plus dollars to fund his retirement. He wants this fund to last at least 10 years. Assuming Simon leaves the unused portion of his investment in this account (term deposit at 7%), explain why Simon will have access to much more than $100,000 per year.

It may come as a surprise to some students, but Simon will be able to receive in excess of $3,300 per week for 10 years before his annuity runs out. That's equivalent to $172,145.24 per year! Of course this doesn't allow for CPI increases or fees, charges and taxes!

TEXAS INSTRUMENTS