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| **Math Objectives**   * Students will explore the family of exponential functions of the form *f* (*x*)  *c* ·*bx**a* and be able to describe the effect of each parameter on the graph of *y*  *f* (*x*). * Students will be able to determine the equation that corresponds to the graph of an exponential function. * Students will understand that a horizontal translation and a vertical stretch of the graph of an exponential function are essentially the same. * Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice). * Students will look for and make use of structure (CCSS Mathematical Practice).   **Vocabulary**   * exponential function * parameter * translation * reflection * vertical dilation   **About the Lesson**   * This lesson involves the family of exponential functions of the form *f* (*x*)  *c* ·*bx**a*. * As a result students will:   + Manipulate sliders, and observe the effect on the graph of the corresponding exponential function.   + Conjecture and draw conclusions about the effect of each parameter on the graph of the exponential function.   + Compare horizontal translation and vertical dilations and manipulate equations to demonstrate they are the same.   + Match specific exponential functions with their corresponding graphs.   **TI-Nspire™ Navigator™ System**   * Transfer a File. * Use Screen Capture to examine patterns that emerge. * Use Live Presenter to demonstrate. * Use Quick Poll to assess students’ understanding. | **TI-Nspire™ Technology Skills:**   * Download a TI-Nspire document * Open a document * Move between pages * Grab and drag a point   **Tech Tips:**   * Make sure the font size on your TI-Nspire handhelds is set to Medium. * You can hide the function   entry line by pressing ctrl G.  **Lesson Files:**  *Student Activity*  Transformations\_of\_Exponential  \_Functions\_Student.pdf Transformations\_of\_Exponential  \_Functions\_Student.doc  *TI-Nspire document*  Transformations\_of\_Exponential  \_Functions.tns  Visit [www.mathnspired.com](http://www.mathnspired.com/) for lesson updates and tech tip videos. |

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| **Discussion Points and Possible Answers** |
| **Tech Tip:** To change a slider setting, right-click in the slider box, and select option 1. Consider changing the (start) value, minimum and/or maximum value, and/or the step size in order to help discover or confirm  the effect of a specific parameter. |

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| **Move to page 2.1.**   1. The graph of *y*  *f* 1(*x*)  *bx* is shown in the right panel. Click   the arrows in the left panel to change the value of *b* , and observe the changes in the graph of *f* 1.   * 1. Explain why for every value of *b*, the graph of *f* 1 passes through the point (0,1).   **Sample Answers:** The graph of *y*  *f* 1(*x*)  *bx* passes through the point (0,1) for all values of *b*  0  because *f* 1(0)  *b*0  1. The *y*-intercept of the graph of  *f* 1 is 1. |  |
| b. For *b*  1, describe the graph of *y*  *f* 1(*x*)  *bx*.  **Sample Answers:** The graph is above the *x* -axis and is always increasing. As *x* takes on smaller and smaller negative values ( 10, 100, 1000, ), the values of *f* 1 get closer to 0.  In more precise mathematical language, we would say as *x* decreases without bound, *bx*  approaches 0. As *x* gets larger and larger (10, 100, 1000, ) the values of *f* 1 get larger and larger. In more precise mathematical language, as *x* increases without bound, *bx* also increases without bound. As *b* gets larger, the graph becomes steeper, or increases more rapidly. As *b*  gets closer to 1, the graph becomes less steep approaching the graph of the line *y*  1. | |
| c. For 0  *b*  1, describe the graph of *y*  *f* 1(*x*)  *bx*.  **Sample Answers:** The graph is above the *x* -axis and is always decreasing. As *x* gets smaller and smaller ( 10, 100, 1000, ) the values of *f* 1 increase without bound. As *x* gets larger and larger (increases without bound), the values of *f* 1 get smaller and approach 0. As *b*  gets closer to 0, the graph becomes steeper. As *b* gets closer to 1, the graph becomes less steep and approaches the graph of the line *y*  1. | |

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| **Teacher Tip:** Teachers might need to remind students that a negative exponent inverts the fraction *b*. The reciprocal of a fraction between 0 and  1 is a number greater than 1. | |
| d. Find the domain and range of function *f* 1(*x*)  *bx*.  **Answer:** The domain is all real numbers, and the range is all positive real numbers: (0,). | |
| e. Does the graph of *y*  *bx* intersect the *x* -axis? Explain why or why not.  **Answer:** For *b*  1: as *x* decreases without bound, the graph of *y*  *bx* approaches the  *x* -axis but never touches it. In limit notation, For 0  *b* 1: as *x* increases without bound, the graph of *y*  *bx* approaches the *x* -axis but never touches it. . In limit notation, The *x* -axis, the line *y*  0, is a horizontal asymptote to the graph of *y*  *bx*. | |
| **Tech Tip:** The limited resolution on the handheld screen might result in a graph that appears to intersect the *x* -axis. This presents an opportunity to trace the graph and/or create a table of values to show that values of the  function are small, but not equal to zero. | |
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| **Teacher Tip:** The slider for the variable *b* is set to minimized, style:  vertical, and initially set such that it includes the value 1. Most definitions of an exponential function stipulate *b*  1. | |
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| **TI-Nspire Navigator Opportunity: *Screen Capture and Quick Poll***  **See Note 1 at the end of this lesson.** | |
| **Move to page 3.1.**   1. The graph of *y*  *f* 1(*x*)  *bx**a* is shown in the right panel. For a specific value of *b* , click the arrows to change the value of *a*   and observe the changes in the graph of *f* 1. Repeat this process for other values of *b*.   * 1. Describe the effect of the parameter *a* on the graph of   *y*  *bx**a*. Discuss the effects of both positive and negative values of *a*. |  |

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| **Answer:** For *a*  0, the graph of *y*  *bx* is translated horizontally, or moved, left *a* units. For *a*  0, the graph of *y*  *bx* is translated right *a* units. | | | | | | |
| **Teacher Tip:** This left/right translation occurs for any value of *b*.  Horizontal translations of the graph of an exponential function are difficult to recognize because students often focus on the *y* -intercept and vertical shifts. Emphasize that the horizontal asymptote ( *y*  0 ) did not change  (move up or down) which would have happened if there were a vertical shift in the graph. | | | | | | |
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| **TI-Nspire Navigator Opportunity: *Screen Capture and Quick Poll***  **See Note 1 at the end of this lesson.** | | | | | | |
| **Move to page 4.1.**   1. The graph of *y*  *f* 1(*x*)  *c* ·*bx**a* is shown in the right panel.   For specific values of *a* and *b* , click the arrows to change the value of *c* , and observe the changes in the graph of *f* 1.  Repeat this process for other values of *a* and *b*.   * 1. Describe the effect of the parameter *c* on the graph of *y*  *c* ·*bx**a*. Discuss the effects of both positive and negative values of *c*.   **Answer:** The graph has a vertical dilation. For | *c* | 1, the graph of *y*  *bx**a* is stretched vertically. For | *c* | 1, the graph of *y*  *bx**a* is compressed vertically. If *c*  0, the graph is reflected across the *x* -axis. | | | | |  | |
| **TI-Nspire Navigator Opportunity: *Screen Capture and Quick Poll***  **See Note 1 at the end of this lesson.** | | | | | | |
| **Move to page 5.1.**   1. Display the graphs of *y*  *f* 1(*x*)  3*x*2 and   *y*  *f* 2(*x*)  9 ·3*x*.   * 1. Describe the similarities between these two graphs. Use the properties of exponents to justify your answer.   **Answer:** The graphs of these two exponential functions are the same. *f* 1(*x*)  3*x*2  3*x* ·32  9 ·3*x*  *f* 2(*x*). | | | | |  | |
| * 1. Insert a new problem, and display the graph of  Use the properties of exponents to find a function of the form  such that the graphs of  and  are the same. Verify your answer.   **Answer:**  The graphs of  and  are the same. | | | | | | |
| * 1. Use your answers to parts (a) and (b) to explain the relationship between a horizontal translation and a vertical dilation of the graph of an exponential function.     **Answer:** A horizontal translation and a vertical dilation of the graph of an exponential function are essentially the same. Consider the following expression to show this analytically.    This demonstrates that any horizontal translation can also be considered a vertical stretch. | | | | | | |
| 1. Without using your calculator, match each equation with its corresponding graph. Check your answers by graphing each function on your calculator. 2. (b) 3. (d) 4. (f)   Note: The function in part (e) is the natural exponential function and involves the number | | | | | | |
| (i) | Description: exponential_functions_5_1b | (ii) | Description: Transformations_of_Exponential_Functions_5_1e | |

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| (iii) | Description: Transformations_of_Exponential_Functions_5_1a | (iv) | Description: Transformations_of_Exponential_Functions_5_1c |
| (v) | Description: Transformations_of_Exponential_Functions_5_1d | (vi) | Description: exponential_functions_5_1f |

**Answers:** (a)  (iii) (b)  (i) (c)  (iv) (d)  (v) (e)  (ii) (f)  (vi).

# Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to:

* Graph and analyze an exponential function of the form
* Explain the concepts of dilation and translation.

*f* (*x*)  *c* ·*bx**a*.

# TI-Nspire Navigator

### Note 1

**Name of Feature: Screen Capture and Quick Poll**

Use Screen Capture to compare student graphs for various values of each parameter.

A Quick Poll can be given at several points during this lesson. It can be useful to save the results and show a Class Analysis.

Sample multiple choice questions:

For *b*  1, how many times does the graph of

### 0 

1. 1
2. 2
3. Infinitely many

*y*  *bx*

cross the x-axis?

How does the graph of

*y*  2*x*5 compare with the graph of

*y*  2*x*.

1. Translated 5 units to the right.

### Translated 5 units to the left. 

1. Shifted five units up.
2. Shifted 5 units down.

For *c*  1,

how does the graph of

*y*  *c* ·3*x* compare with the graph of

*y*  *c* ·3*x* ?

1. Wider
2. Stretched

### Reflected 

1. Same