Metallic Numbers

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Student Activity

7 8 9 **10** **11** 12

TI-84PlusCE™ Investigation Student 30 min

Introduction

The famous Fibonacci sequence 1, 1, 2, 3, 5, 8 ... involves the recursive sequence definition: $t_{n+2} = t_n + t_{n+1}$.

The ratio between consecutive terms for the Fibonacci sequence as $n \to \infty$ is known as the Golden Ratio:

$$\text{Golden Ratio: } \lim_{n \to \infty} \frac{t_{n+1}}{t_n} = \phi$$

In this investigation you will explore a small variation on the Fibonacci sequence: $t_{n+2} = t_n + a t_{n+1}$ where a is a natural number. In this investigation these variants on the Fibonacci sequence will be referred to as "Levels", for example Fibonacci Level 2 means that $a = 2$ in the recursive definition above. The original Fibonacci sequence is therefore Fibonacci Level 1 with $a = 1$.

Fibonacci Level 2: In search of the Silver Ratio

This sequence starts as: 1, 1, 3, 7, 17, 41, 99 …

Each successive term is equal to "the previous two terms plus another helping of the previous term." This can be expressed more succinctly using mathematical notation as:

$$t_{n+2} = t_n + 2t_{n+1}$$

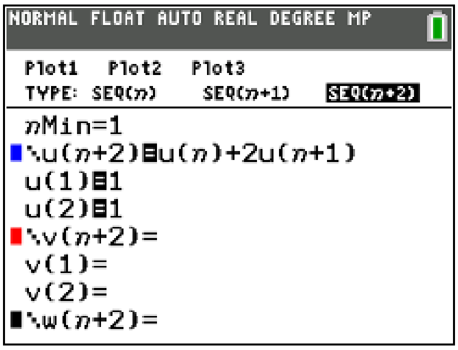
The first two numbers can still be set as 1 and 1.

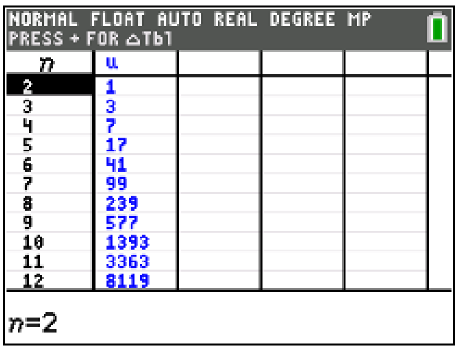
Make sure your calculator is set in 'sequence' mode: [MODE] > Seq

Use the y = editor to select the sequence type: SEQ(n+2)) shown.

Use [ALPHA] and 'u' (located above the 7 key) to reference terms.

To check the sequence, generate a table of values: [2nd] [GRAPH]





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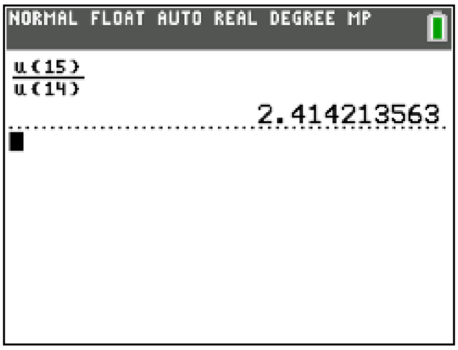
Calculate the ratio between consecutive terms.

Use the 'u' (located above the 7 key) to reference each term.



Question: 1.

Explore the ratio between consecutive Fibonacci Level 2 terms, this is called the 'Silver' ratio.

Calculator Tip!

It is possible to generate an entire list of ratios directly on the calculator home screen. Start by storing the numbers from 1 to 15 in L_1. The Sequence command located in the List > Op menu will help.

seq(x,x,1,15) $\rightarrow L_1$

With the Fibonacci Level 2 sequence defined in *u*, generate an entire list of ratios:

$$\frac{u(L_1+1)}{u(L_1)}$$

Question: 2.

Change the first two terms in the Fibonacci Level 2 sequence and check to see if this changes the long term value of the ratio between consecutive terms.

Question: 3.

Let x represent any term in the sequence and y the next term.

a) Explain the two formulas below:

$$r_n = \frac{y}{x} \quad \text{and} \quad r_{n+1} = \frac{2y+x}{y}$$

b) Assuming the ratio between consecutive terms is approximately equal as $n \rightarrow \infty$ determine the value of the ratio.

Fibonacci Level 3: In search of the Bronze Ratio

The bronze ratio refers to the ratio between consecutive terms of the level 3 Fibonacci sequence. The general formula for the sequence $t_{n+2} = t_n + at_{n+1}$ therefore becomes: $t_{n+2} = t_n + 3t_{n+1}$

Question: 4.

Generate the first 50 terms of the level 3 sequence and store them in L_3, explore the ratio between consecutive terms as n increases.

Question: 5.

Set up two formulas similar to those from Question 3 and hence determine the exact value for the bronze ratio.

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Fibonacci Level *n*: The Metallic Ratios

The general term for the ratio between consecutive terms for $t_{n+2} = t_n + at_{n+1}$ is referred to as a Metallic ratio.

Question: 6.

Determine an expression for the general form of the Metallic ratios. Check your answer using $a=1$, $a=2$ and $a=3$.

Question: 7.

For the golden ratio (ϕ) the following relationships hold:

$\phi = \frac{1}{\phi} + 1$	$\phi^2 = \phi + 1$	$\phi^1 + \phi^2 = \phi^3$

Do any of the above relationships hold for the silver or bronze ratio?

Question: 8.

Calculate the approximate value for each of the following and comment on your finding as the quantity of 'embedded' fractions increases.

a) $1+\cfrac{1}{1+1}$

b) $1+\cfrac{1}{1+\cfrac{1}{1+1}}$

c) $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}$

d) $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}}$ (In this case add as many fractions as possible)

Question: 9.

Calculate the approximate value for the following 'embedded' fraction and comment on the result.

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots}}}}}$$

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