Teacher Notes

Activity 16

Accumulation Functions

Objectives

- Examine functions defined by a definite integral
- Understand the foundation of the Fundamental Theorem of Calculus

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 50 minutes

Abstract

In this activity, students create tables and graphs for functions of the form

$$f_a(x) = \int_a^x 1.5\,dt$$

The graphing handheld is used to produce a scatter plot. This activity is meant to precede and foreshadow the Fundamental Theorem of Calculus activity. Question **13** has students directly anticipate the Fundamental Theorem of Calculus.

Management Tips and Hints

Prerequisites

Students should know the notation for definite integrals and understand these basic properties:

$$\int_a^a f(t)dt = 0$$

If $f(t) > 0$ for $a \le t \le x$, then

$$\int_a^x f(t)dt > 0$$

If $f(t) > 0$ for $x \le t \le a$, then

$$\int_a^x f(t)dt < 0$$

Evidence of Learning

Students will

- be able to evaluate functions of the form $g(x) = \int_a^x k\,dt$, where k is a constant.
- recognize that the graphs of functions of the form $g(x) = \int_a^x k\,dt$, are parallel lines.

Common Student Errors/Misconceptions

This is likely the first experience students will have with a function defined with the independent variable as a limit of integration. A discussion of the integral of a constant velocity function from 0 to *t* could help students relate this to a familiar concept. For example, if

$$s(t) = \int_0^t 50\,du$$

then $s(2) = 100$ is the distance traveled by a car traveling at a constant velocity of 50 mph between times $t = 0$ and $t = 2$. Discuss the meaning of $s(t)$ for any time t.

Activity Solutions

1. 0

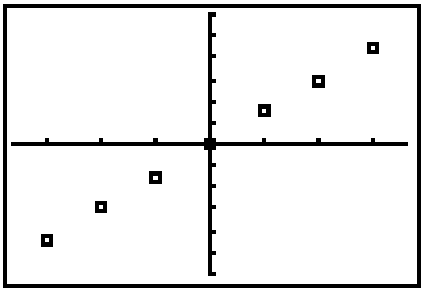
2. 1.5

3. −1.5

4.

x	$f_0(x) = \int_0^x 1.5\,dt$
0	0.0
1	1.5
2	3.0
3	4.5
−1	−1.5
−2	−3.0
−3	−4.5

5.



6. 0

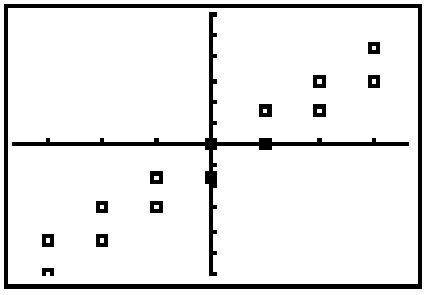
7. 1.5

8. −1.5

9.

x	$f_1(x) = \int_1^x 1.5\,dt$
0	-1.5
1	0.0
2	1.5
3	3.0
-1	-3.0
-2	-4.5
-3	-6.0

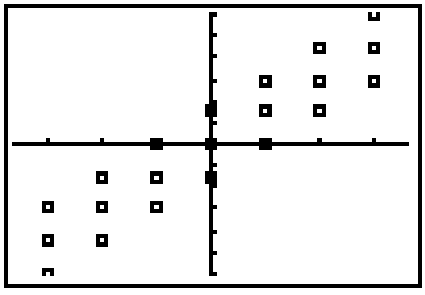
10.



11.

x	$f_{-1}(x) = \int_{-1}^x 1.5\,dt$
0	1.5
1	3.0
2	4.5
3	6.0
-1	0.0
-2	-1.5
-3	-3.0

12.



13. Each is contained in a line that has a slope of 1.5.