Case File 6

Measuring Momentum: Using distance moved after impact to estimate velocity

Explore how the speed of an impacting vehicle causes a stationary object to move.

Police Report

Last Tuesday night, police officers were dispatched to the remote icy intersection of Elm Road and Winding Way for a routine collision investigation. The driver of a car seems to have lost control of his vehicle on the ice and crashed into a black van sitting idle on the side of the road.

Investigators took some measurements at the crime scene in order to determine how fast the car was going before it hit the van. Since the car skidded on an icy road, there were no tire tracks to measure to determine its speed. However, the van was parked on a sandy shoulder, and investigators were able to measure the distance it was pushed when the car hit it.

The marks in the sand show that the van was pushed 1.6 meters when it was struck by the car. According to the car manufacturer, the car has a mass of 1000 kg.

Forensics Objective

- establish a relationship between the momentum of a vehicle and the distance a stationary object moves when the vehicle hits it

Science and Mathematics Objectives

- accurately gather data from a collision of a vehicle with a stationary object
- establish a relationship between the distance an object moves after a collision with a vehicle and the momentum of the vehicle

Materials (for each group)

- TI-83/TI-84 Plus™ Family
- Vernier EasyData™ application
- Vernier EasyLink™
- Dual-Range Force Sensor
- Calculator-Based Ranger™ 2 (CBR™ 2)
- clamp or heavy tape
- 1.5 m (or longer) ramp of strong cardboard or wood
- meterstick
- 25 cm (or higher) support for ramp
- masking tape
- 450–500 g vehicle about 7 cm tall
- book, 5–7 cm thick
- book, 2 cm thick
- string or wide rubber band

Procedure

1. Set the switch on the Dual-Range Force Sensor to ±10 N. Plug the force sensor into EasyLink, and plug EasyLink into the calculator. The calculator should automatically turn on and take you to the Main screen for the force sensor.

2. Hold the force sensor vertically with the hook pointing down. Hang a rubber band or string from the hook.

3. Select Setup and choose option **7: Zero**. When the weight has stabilized, select Zero (not 0). This will zero the sensor for weighing the objects with the rubber band or string attached.

TIP! At the bottom of the Main screen are five options (File, Setup, Start, Graph, and Quit). Each of these options can be selected by pressing the calculator key located below it (Y=, WINDOW, ZOOM, TRACE, or GRAPH).

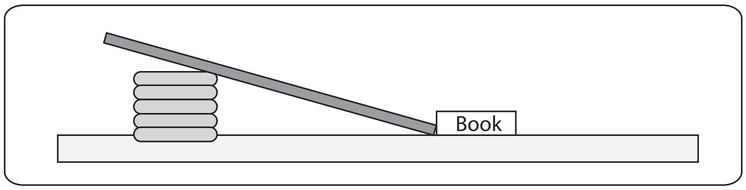
4. Calculate the mass of the vehicle that you will be using to collide with the books.
   a) Hang the vehicle from the force sensor with the string or rubber band. The weight of the vehicle, in newtons, will be displayed on the top of the screen. Record this number in your Evidence Record.
   b) Convert the weight to mass, using the equation below, and record it in your Evidence Record.

$$\text{Mass in kilograms} = \frac{\text{Weight in newtons}}{\text{Acceleration due to gravity } (9.8\,\text{m/sec}^2)}$$

5. Weigh and calculate the mass of each of the books, and record the numbers in your Evidence Record.

6. Unplug the force sensor from the calculator's USB port.

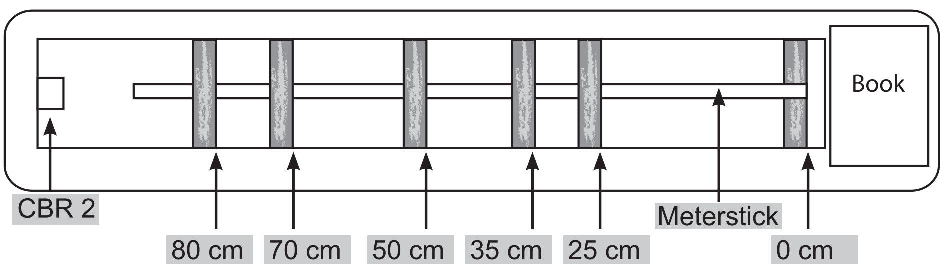
7. Set up the ramp as shown in the figure below. Place the heavier (thicker) of the two books at the base of the ramp.



View from the side

8. With the book at the bottom of the ramp, place the vehicle at the bottom of the ramp against the book. Apply a strip of tape to the ramp at the location of the front wheels of the vehicle. The tape should be about 10 cm long and parallel to the bottom edge of the ramp.

9. Place a meterstick in the center of the ramp with the 0 at the bottom edge of the tape you applied in step 8. Tape it securely to the ramp at the following distances: 25, 35, 50, 70, and 80 cm from the bottom edge of the tape, as shown in the figure below. The meterstick will provide an additional support for a cardboard ramp, a guide for the vehicle, and a way to measure the distance the vehicle will travel down the ramp.



View from above

10. Firmly attach the CBR 2 cable to the USB port of your calculator. This will automatically start the EasyData App. Distance data will be displayed at the top of the screen.

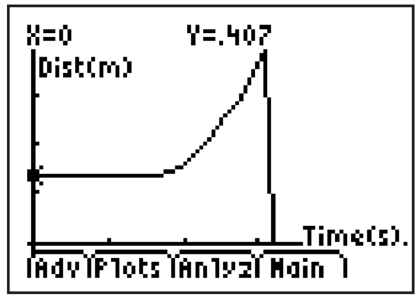
11. Change the timed-experiment parameters to record one sample every 0.05 seconds for 2 seconds.
    a) Select **Setup** from the Main screen.
    b) Select option **2: Time Graph**, and then select **Edit**.
    c) Press CLEAR and then type **0.05** as the sample interval. Select **Next**.
    d) Press CLEAR and then type **40** for the number of samples. Select **Next**.
    e) When you have confirmed that the time graph settings are correct (0.05-second sample interval, 40 samples, 2-second experiment length), select **OK**.

12. Place the CBR 2 at the top of the ramp. There should be at least 25 cm between the CBR 2 and the rear of the vehicle when the vehicle is placed with its front wheels at the 80 cm mark.

13. Place the vehicle on the ramp so that it straddles the meterstick and has its front wheels at the 80 cm mark.

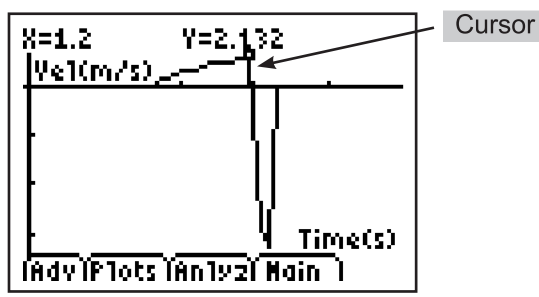
14. Select [Start] to begin data collection. (Note: You may also have to select [OK] to overwrite old data.) Release the vehicle about half a second after you hear the rapid clicking sound made by the CBR 2.

15. When the vehicle has collided with the book and the CBR 2 has stopped clicking, the calculator will plot a distance vs. time graph on the screen. It should look similar to the one shown below. (Note: Do not be confused by any unusual distance readings that may show up on the graph for times after the vehicle has collided with the book. Focus on the data collected only up to the point that the vehicle struck the book.)



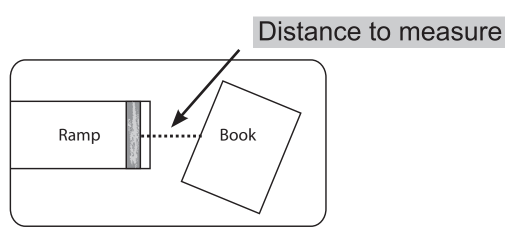
Determine the vehicle's velocity when it struck the book.

a) Select [Plots] from the Main screen and choose option **2:Vel(m/s) vs Time**. Your velocity vs. time graph should be similar to the graph below. The velocity when the vehicle was at the end of the ramp will be the maximum point on the graph. (Note: Again, do not be confused by any unusual velocity readings on this graph for times after the vehicle collided with the book.)



b) Move your cursor to the point corresponding to the time when the vehicle struck the book. Record the velocity of the vehicle at this point. The velocity is the *y*-value shown at the top of the screen. Record this velocity in your Evidence Record. If your graph does not have a clear maximum point, repeat steps 13–15. You may need to make several runs to get good results.

16. Measure the distance the book was pushed by its collision with the vehicle. Enter that distance in *meters* into your Evidence Record. If the book was not pushed parallel to the ramp, measure the distance from its starting point to the edge of the book along its centerline, as shown in the figure below. Once you have measured the distance, return the book to its original position at the base of the ramp.



Distance to measure, Book, Ramp

17. Repeat steps 13–16 twice more. Calculate the average of the results and record it in the Evidence Record.

18. Repeat steps 13–17 for the other distances you have marked off on your ramp (70, 50, 35, and 25 cm).

19. Repeat steps 13–18 for the lighter (thinner) book.

20. Calculate the average momentum of the vehicle at impact for each average distance the book moved. Do this by multiplying the average speed (velocity) of the vehicle by the mass of the vehicle. Record the average momentums in the Evidence Record.

21. Enter the momentum and distance data for the *heavy* book into lists in the calculator.
    a) Exit EasyData by selecting Quit then OK.
    b) Press STAT ENTER to open the list editor. If any of the lists already contain data, use the arrow keys to highlight the title of the list and press CLEAR ENTER to erase all the old data in the list.
    c) In the first list, L1, enter the average momentum of the vehicle that you calculated for each incline distance (distance vehicle traveled).
    d) Use the arrow to move over to the second list, L2, and enter the average distance that the heavy book moved for each incline distance.

22. Plot a graph of the distance the heavy book moved versus the momentum of the vehicle at impact.
    a) Press 2nd Y= to enter the **STAT PLOT** menu.
    b) Press ENTER to select **Plot 1**. Use the arrow keys to highlight **On**, and then press ENTER to select.
    c) Use the arrow keys to highlight the first type of graph, a dot (scatter) plot, and then press ENTER to select.
    d) Use the arrow keys to highlight the entry next to **Xlist:**. Press 2nd 1 to select list **L1**. This will tell the calculator to plot momentum (list L1) on the *x*-axis.
    e) Use the arrow keys to highlight the entry next to **Xlist:**. Press 2nd 2 to select list **L2**. This will tell the calculator to plot distance (list L2) on the *y*-axis.
    f) Use the arrow keys to highlight the entry next to **Mark:**. Highlight the first mark and press ENTER to select.
    g) Press ZOOM and select option **9↓:ZoomStat** to automatically scale your graph.
    h) Your screen should now show a graph of the vehicle's momentum on the *x*-axis and the distance it pushed the book on the *y*-axis.

23. You will be using the calculator's LinReg program to determine the equation for the straight line that fits your data best. A linear regression is a mathematical formula that can be used to do this. (Note: It is possible to calculate the equation by hand, but it takes a long time and is tedious. Your calculator has a program called LinReg that will calculate it quickly.) Turn on the calculator's **Diagnostic** function. This will tell the calculator to determine how well the line fits the data, in addition to calculating the equation for the line.
    a) Go to the calculator's function catalog by pressing 2nd 0.
    b) Use the arrow keys to scroll down to the **DiagnosticOn** option, and then press ENTER ENTER to select it and turn it on.

24. Use the LinReg function to perform the linear regression and store the resulting equation in variable **Y1**:
    a) Press STAT, highlight **CALC**, and choose option **4: LinReg(ax + b)**. The Home screen will read **LinReg(ax + b)**. (Do *not* press ENTER yet.)
    b) You need to tell the calculator where your data are and where to store the final equation. Press 2nd 1 , 2nd 2 , to tell the calculator that lists L1 and L2 contain the data that you want to fit a line to. (Do *not* press ENTER yet.)

c) To indicate that you want to store the equation in variable Y1, press [VARS] [▶] to select **Y-VARS**. Press [ENTER] [ENTER] to tell the calculator to store the equation in the **Y1** variable. The Main screen should now read **LinReg(ax+b) L1,L2,Y1**.

25. Now press [ENTER] to execute the LinReg function. This function calculates the equation for the straight line that fits through your data best. The screen will display the values for a and b that make the linear equation fit the data. The r^2 value tells you how well the line fits the data. An r^2 value that is close to 1 means that the line fits the data very well. Fill in the LinReg (Heavy Book) table in the Evidence Record.

26. Press [GRAPH]. You should see your graph and a best fit line being drawn through the points. Sketch the graph of distance vs. momentum data in the Evidence Record.

27. Press [2nd] [MODE] to quit the graph screen and go back to the Home screen. Repeat steps 21–26 for the *light* book.

Name: ______________________________________
Date: ______________________________________

Evidence Record

Weight of vehicle ______________ N
Mass of vehicle ______________ kg

For the Heavy Book

Weight of book ______________ N
Mass of book ______________ kg

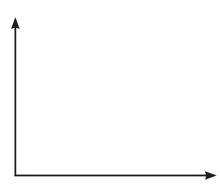
Distance Vehicle Traveled Before Colliding with Book (m)	Velocity of Vehicle (m/sec)	Distance Book Moved (m)
0.8		
0.8		
0.8		
0.7		
0.7		
0.7		
0.5		
0.5		
0.5		
0.35		
0.35		
0.35		
0.25		
0.25		
0.25		

Distance Vehicle Traveled Before Colliding with Book (m)	Average Velocity of Vehicle (m/sec)	Average Distance Book Moved (m)	Average Momentum of Vehicle at Impact (kg·m/sec)
0.8			
0.7			
0.5			
0.35			
0.25			

LinReg (Heavy Book)

y	*ax* + b
a	
b	
r^2	

Sketch of graph of distance vs. momentum data



For the Light Book

Weight of book _______________ N
Mass of book _______________ kg

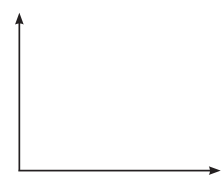
Distance Vehicle Traveled Before Colliding with Book (m)	Velocity of Vehicle (m/sec)	Distance Book Moved (m)
0.8		
0.8		
0.8		
0.7		
0.7		
0.7		
0.5		
0.5		
0.5		
0.35		
0.35		
0.35		
0.25		
0.25		
0.25		

Distance Vehicle Traveled Before Colliding with Book (m)	Average Velocity of Vehicle (m/sec)	Average Distance Book Moved (m)	Average Momentum of Vehicle at Impact (kg·m/sec)
0.8			
0.7			
0.5			
0.35			
0.25			

LinReg (Light Book)

y	$ax + b$
a	
b	
r^2	

Sketch of graph of distance vs. momentum data



Case Analysis

1. Is the linear regression a good fit to the data for the heavy book? For the light book? Explain.
2. For the *heavy* book, write the equation for the distance vs. momentum graph. Use $y = ax + b$ and values from step 25. The y is distance the book moved, and x is momentum at impact.
3. Using your equation, calculate how far the book would move if its momentum at impact were 0.7 kg·m/sec. Show your work. Don't forget that your answer has units.
4. If the book moved 0.2 m after impact, what would the momentum of the vehicle have been when it hit the book? Show your work.
5. Repeat questions 3 and 4 for the *light* book.
6. The equation for the accident scene is $y = 0.00245x - 37.8$, where y is distance the van moved (in meters) and x is momentum of the car at impact (in kg·m/s). What was the car's momentum just before it hit the van? (Hint: Rearrange the equation to solve for x.) Show your work.
7. What was the impact velocity of the car in meters per second? Show your work. (Hint: Rearrange the momentum equation, $p = m \cdot v$ where p is momentum, m is mass, and v is velocity. Then substitute for momentum, from question 6, and mass of the car, 1000 kg.)
8. If the speed limit on the road was 50 mph, was the car speeding just before it hit the van? Show your work. (Hint: Multiply velocity in meters per second, from the last question, by 2.24.)

Case File 6
Measuring Momentum: Using momentum to determine intent

Teacher Notes

Teaching time: one or two class periods

This lab introduces the concept of momentum as it applies to vehicle collisions.

Tips

- Test the setup to be sure that the vehicle will move the books a measurable distance from all of the designated points on the ramp. If this does not happen, add some weight to the vehicle or use a lighter book.
- If a Dual-Range Force Sensor is not available, use a pan balance or spring scale to find the mass of the books and vehicle directly.
- When taping the meterstick to the ramp, make sure that the tape does not interfere with the movement of the vehicle's wheels.

Lab Preparation

Materials
- A 1.5 m piece of corrugated board can very likely be found at a furniture or grocery store. If you are unable to locate a piece of corrugated board, a thin piece of paneling cut to 1.5 m × 40 cm will also work as a ramp.
- If you are using a toy truck, two metersticks taped to the ramp may be a better guide for the truck.
- If you have access to a Vernier® Dynamics System, the cart from that system, with a 3 × 5 inch card taped to the rear of the cart (to reflect the sound waves), works well.

Setup Information
- It is best to run this activity on a smooth surface, such as an uncarpeted floor or smooth table. The lower the coefficient of friction between the book and the surface, the better the data.
- Collecting good data for the velocity of the vehicle and the distance the book moves (it may move on the diagonal) can take a lot of time. If you have two ramps and similar vehicles, it may be a good idea to have two teams of students working. One can start with the lighter book and one with the heavier one. If class time is available, have the teams switch books at the end. Otherwise, have them share data.
- Ideally, the values for maximum speed and momentum at impact should be the same for the two books because the release points are identical. This rarely happens, but these values should be close to each other if the data are acceptable.

Background Information

The momentum of a vehicle depends upon the mass of the vehicle and its velocity. The equation is $p = m \cdot v$, where p is the momentum, m is the mass, and v is the velocity.

If you make several assumptions, the distance that an object slides when it is hit by a moving object should be related to the square of the velocity, rather than to the velocity in the linear relationship modeled here. If students were able to make extremely accurate measurements of sliding distance and velocity, they would see the square relationship. However, because the books tend to slide in different directions and accurate velocity measurements are difficult to obtain, the students will probably see a linear, rather than a square, relationship. You may want to discuss this with your students.

Modifications

If time is short, have students skip step 17.

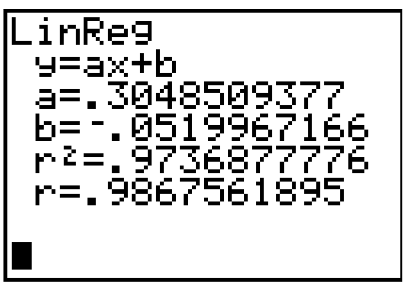
Sample Data

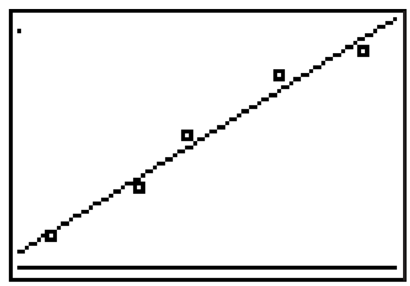
Weight of vehicle 2.156 N
Mass of vehicle 0.220 kg

For the Heavy Book

Weight of book 6.407 N
Mass of book 0.654 kg

Distance Vehicle Traveled Before Colliding with Book (m)	Average Velocity of Vehicle (m/sec)	Average Distance Book Moved (m)	Average Momentum of Vehicle at Impact (kg·m/sec)
0.8	2.336	0.1	0.514
0.7	2.040	0.09	0.449
0.5	1.717	0.068	0.378
0.35	1.551	0.048	0.341
0.25	1.243	0.030	0.273

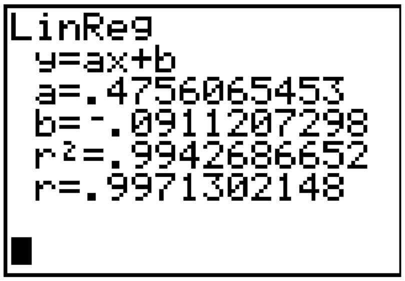


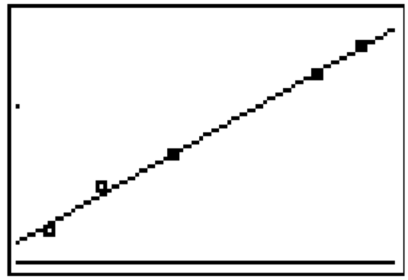


For the Light Book

Weight of book 5.2828 N
Mass of book 0.483 kg

Distance Vehicle Traveled Before Colliding with Book (m)	Average Velocity of Vehicle (m/sec)	Average Distance Book Moved (m)	Average Momentum of Vehicle at Impact (kg·m/sec)
0.8	2.201	0.14	0.484
0.7	2.074	0.124	0.456
0.5	1.652	0.081	0.363
0.35	1.438	0.064	0.316
0.25	1.285	0.040	0.283





Case Analysis Answers

1. Is the linear regression a good fit to the data for the heavy book? For the light book? Explain.
   Since r and r^2 are close to 1, a linear relationship between momentum and distance is a good model for the data. For the sample data, the data for the light book are better than those for the heavy book.
2. For the *heavy* book, write the equation for the distance vs. momentum graph. Use $y = ax + b$ and values from step 25. The y is distance the book moved, and x is momentum at impact.
   Answers will vary. For the sample data, the equation is y = 0.305x − 0.052.
3. Using your equation, calculate how far the book would move if its momentum at impact were 0.7 kg·m/sec. Show your work. Don't forget that your answer has units.
   Answers will vary. For the sample data, y = 0.305 × 0.7 − 0.052 = 0.162 m, so the book would move 0.162 m.
4. If the book moved 0.2 m after impact, what would the momentum of the vehicle have been when it hit the book? Show your work.
   Distance = 0.20 = 0.305x − 0.052, so the momentum would be 0.826 kg·m/s.
5. Repeat questions 3 and 4 for the *light* book.
   Answers will vary. For the sample data, y = 0.476 × 0.7 − 0.091 = 0.242 m, so the book would move 0.242 m; if distance = 0.20 = 0.476x − 0.091, the momentum would be 0.611 kg·m/s.
6. The equation for the accident scene is $y = 0.00245x - 37.8$, where y is distance the van moved (in meters) and x is momentum of the car at impact (in kg·m/s). What was the car's momentum just before it hit the van? (Hint: Rearrange the equation to solve for x.) Show your work.

   y =1.6 = 0.00245x − 37.8
   39.4 = 0.00245x
   momentum, x = 16,082 kg·m/s
7. What was the impact velocity of the car in meters per second? Show your work. (Hint: Rearrange the momentum equation, $p = m \cdot v$ where p is momentum, m is mass, and v is velocity. Then substitute for momentum, from question 6, and mass of the car, 1000 kg.)

   momentum = 16,082 = 1000v
   velocity, v = 16.08 m/s
8. If the speed limit on the road was 50 mph, was the car speeding just before it hit the van? Show your work. (Hint: Multiply velocity in meters per second, from the last question, by 2.24.)

   velocity, v = 16.08 × 2.24 = 36.0
   The car was not speeding because its velocity at impact was 36.0 mph.