



Math Objectives

- Students will explore the relationship between rational functions and both vertical and horizontal asymptotes. They will examine these relationships graphically and algebraically.
- Students will be asked to store values as variables on the handheld to help in their exploration and discussions.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Input
- Output
- Vertical Asymptotes
- Horizontal Asymptotes

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 2 Functions:

2.8: (a) The reciprocal function $f(x) = \frac{1}{x}$, $x \neq 0$, its graph and self-inverse nature.

(b) Rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ and their graphs.

(c) Equations of vertical and horizontal asymptotes.

As a result, students will:

- Apply this information to real world situations.

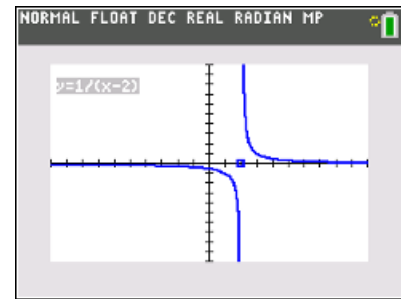
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

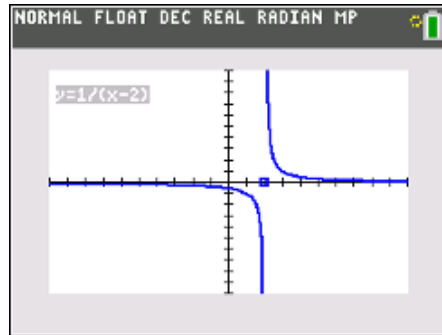
Lesson Files:

Student Activity
 84CE-RationalFunctions-Student.pdf
 84CE-RationalFunctions-Student.doc
 RATIONAL.8XP



TI-84 PLUS CE FAMILY

In this activity, you will explore the properties of functions of the forms $f(x) = \frac{1}{x-a}$ and $f(x) = \frac{ax+b}{cx+d}$ using a program called **RATIONAL** that allows you to dynamically examine these rational functions.



Using the handheld, students will discover, or re-discover, the connection between a rational function, transformations, and both vertical and horizontal asymptotes. To accomplish this, they will be running a program called **RATIONAL**.

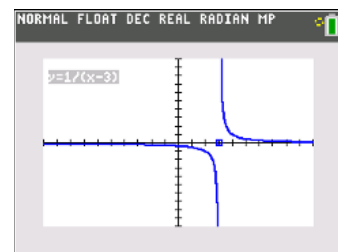
Let's review the idea behind vertical and horizontal asymptotes. A **vertical asymptote** is a vertical line that guides the graph of the function but is not part of it. It can never be crossed by the graph because it occurs at the x-value that is not in the domain of the function. A function may have more than one vertical asymptote. A **horizontal asymptote** is a horizontal line that the graph of a function approaches as the magnitude of the input increases without bound in either a positive or negative direction. A function may cross a horizontal asymptote for finite values of the input.

Problem 1 – Using the Program RATIONAL to Discover Vertical Asymptotes

After the program has been downloaded onto the handheld, and from the home screen, press **prgm**, **1: TI-Basic**, and select the program **RATIONAL**. Press **enter** to run the program.



The program shows the graph of $y = \frac{1}{x-a}$. The value of a is shown by a dot on the x-axis at $(a, 0)$. The starting value of a is 3. Press the left and right arrows to adjust the value of a . Pressing the up or down arrows will exit the program. Notice that as the value of a changes, the equation and graph are updated.





Discussion Questions

1. With a classmate, press the left and right arrows and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the gap between the two curves.

Possible Explanation: As you press the left/right arrow buttons, the curve moves left and right and there seems to be a relationship with the gap between the curves and the a value in the denominator of the function.

2. Find the value of x where the function $y = \frac{1}{x-2}$ is undefined.

Solution: $x = 2$

3. Find the value of x where the function $y = \frac{1}{x+1}$ is undefined.

Solution: $x = -1$

4. Find the value of x where the function $y = \frac{1}{x-a}$ is undefined.

Solution: $x = a$

5. Find the value of x where the function $y = \frac{1}{x-a}$ has a vertical asymptote. With a classmate, come up with a rule to find the vertical asymptote.

Solution: Vertical asymptote at $x = a$. Possible rule: Set the denominator equal to zero and solve for x.

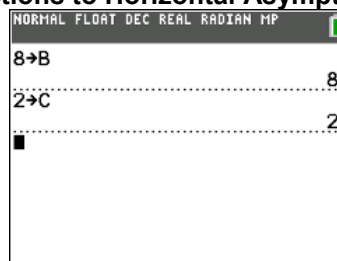
Extension Question:

6. Discuss with a classmate what you notice about what happens horizontally as you move the curve left and right. Write an explanation and share with the class.

Possible Discussion and Explanation: No matter how far left or how far right you move the rational function, the curves seem to approach the x-axis, or $y = 0$. This could possibly represent a horizontal asymptote.

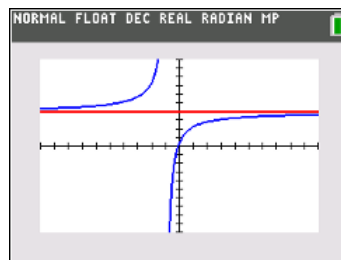
Problem 2 – Using Graphs on the Handheld to Make Connections to Horizontal Asymptotes

On your home screen, store 8 in for the letter B by pressing **8, sto, enter**. Then store 2 in for the letter C by pressing **2, sto, enter**. Go to the **y =** screen and enter the fraction $\frac{b \cdot x + 1}{c \cdot x + 2}$ into **Y₁** and press **enter**.





To make the connection once again to asymptotes, we will add a line. Enter the fraction $\frac{b}{c}$ into Y_2 and press **graph**.



Discussion Questions

1. With a classmate, look at the graph and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the horizontal line.

Possible Explanation: The curve seems to approach the horizontal line and the horizontal line seems to share the same value as the b and c values entered in the equation, or the $\frac{b}{c}$ value of the function.

2. Find the value of y that the function seems to approach as the input values approach both positive and negative infinity when $Y_1 = \frac{bx+1}{cx+2}$.

Solution: $y = \frac{8}{2} = 4$

3. Go back to the **y =** screen and change the equation in Y_1 to be $\frac{cx+1}{bx+2}$ and the fraction in Y_2 to be $\frac{c}{b}$. Look at the graph. Find the value of y that the function seems to approach as the input values approach both positive and negative infinity when $Y_1 = \frac{cx+1}{bx+2}$.

Solution: $y = \frac{2}{8} = \frac{1}{4}$

4. Find the values of y for each function entered into Y_1 from questions 2 and 3 where you think each has a horizontal asymptote. With a classmate, come up with a rule to find the horizontal asymptote.

Solution: Question 1: $y = \frac{b}{c}$

Question 2: $y = \frac{c}{b}$

Possible rule: Since the rational function is made up of a linear function divided by a linear function, the horizontal asymptote is the leading coefficient of the numerator divided by the leading coefficient of the denominator.



Teacher Tip: For this further IB application, in addition to asymptotes, students will be required to find zeros and an inverse function, these topics many need to be reviewed.

Further IB Application

The function f is defined by $f(x) = \frac{8x-8}{3x-6}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$.

Solution: Factor the numerator and denominator to see if there is a hole.

$$f(x) = \frac{8x-8}{3x-6} = \frac{8(x-1)}{3(x-2)}, \text{ factors cannot cancel, no hole.}$$

Set the numerator equal to zero and solve for x .

$$8x - 8 = 0$$

$$x = 1$$

(b) For the graph of $y = f(x)$, write down the equation of

(i) the Vertical Asymptote;

Solution: Set the denominator equal to zero and solve.

$$3x - 6 = 0$$

$$x = 2$$

(ii) the Horizontal Asymptote.

Solution: Since the degrees of the numerator and denominator are equal, divide the leading coefficients.

$$y = \frac{8}{3}$$

(c) (i) Find $f^{-1}(x)$, the inverse function of $f(x)$.

Solution: Switch x and y and solve for y .

$$x = \frac{8y-8}{3y-6} \quad (\text{cross multiply})$$

$$3xy - 6x = 8y - 8 \quad (\text{bring } y \text{ terms to one side})$$

$$3xy - 8y = 6x - 8 \quad (\text{factor out a } y)$$

$$y(3x - 8) = 6x - 8 \quad (\text{solve for } y)$$

$$f^{-1}(x) = y = \frac{6x-8}{3x-8}$$

(ii) Hence, or otherwise, find the value of $f^{-1}(0)$.

Solution: Substitute zero into your answer to part c(i) or use the answer to part (a).

$$f^{-1}(0) = \frac{6(0)-8}{3(0)-8} = \frac{-8}{-8} = 1$$

or

Since $f(1) = 0$, then $f^{-1}(0) = 1$.



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Teacher Tip: Throughout this activity, the students are asked to discuss with classmates and explain how they achieved their answers. This is a wonderful opportunity to create a student led classroom. As you float around the room, listen to what they are saying, add to their discussions, and give them leading questions to see how they respond.

Teacher Tip: Basic understanding of the TI-84 CE is needed to store variables and run programs. It may be a good idea to review these before the activity.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*