Inside Out

Introduction

What shapes have a centre? A circle can be defined as a set of points equidistant from a fixed point, the centre of the circle. So the distance from the centre of the circle is the same to all points on its circumference. The diagonals of a square intersect at a single point, this point is equidistant from each of the square's vertices. An equilateral traingle, just like the square, has a point that is equidistant from its vertices. What about an isosceles or scalene triangle, do they have a similar point?

Teacher Notes:

It is important for students to discuss their ideas about the 'centre' of an shape or object prior to commencing this activity. Most students assume the centre of an object must be contained within the boundaries of the shape, hence the title of the activity, 'inside out'. By way of introduction, drawing a selection of shapes such as circles and regular polygons promotes discussion relating to the centre of an object. Prompt students to discuss whether their centre is the same distance from 'each side' or from 'each vertex'. A Power Point slide show is available with this activity bundle that contains a sequence of shapes to help generate discussion. Most of the shapes are 'regular' with a signficant exception, the last triangle.

Paper folding is another great way for students to start exploring. Each student can start by cutting out an acute-angled triangle. Folding each side such that the vertices are brought together makes a series of perpendicular bisectors that will meet at the 'centre' of the triangle. Students can measure the distance from this point to each of the vertices and comment on their answer. The paper folding activity is relatively static whereas the geometry application on TI-nspire provides a dynamic representation resulting in greater opportunities for exploration, generalisation and finally proof.

Pratical applications can be introduced via images such as a high jumper clearing the high jump bar and global positioning systems. The 'Frosby flop' allows the high jumper's centre of gravity to pass beneath the bar thereby reducing the energy required to clear the jump and hence the understanding that the centre does not have to be located within the boundaries of the shape. Global Positioning Systems use triangulation to help locate an object.

Setting up the Diagram

Press the **Home** key, select **New Document** and insert a **Geometry** Application.
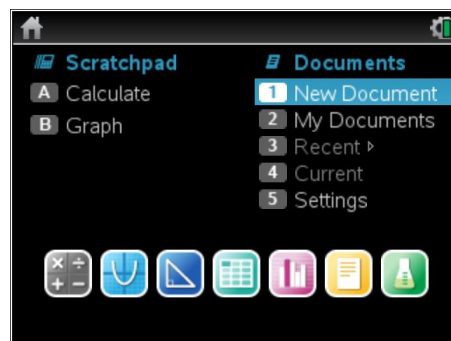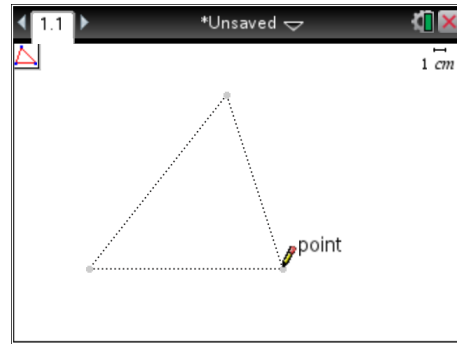
Use the **shapes** menu to construct a **triangle**.

> **[Menu] > Shapes > Triangle**

The mouse changes to a pencil. Make an acute angled triangle that is approximately half the height of the screen. Click on the mouse pad to place the vertices (corners) of the triangle.

> Sample shown opposite.

Use the **construction** menu to draw a **perpendicular bisector** on each side of the triangle.

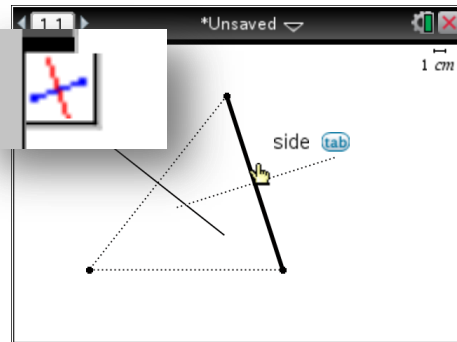> **[Menu] > Construction > Perpendicular Bisector**

Move the mouse over one of the triangle sides, the word 'side' appears as a prompt, the side of the triangle is highlighted and the perpendicular bisector appears dotted.

Click on the side to construct the perpendicular bisector.
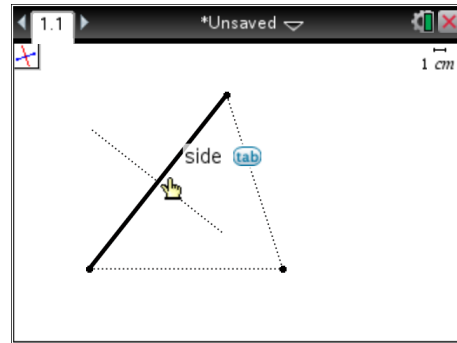
Repeat the previous step for the other two sides of the triangle. Note that the perpendicular bisector tool is still active therefore there is no need to return to the menu, simply click on each of the remaining sides of the triangle to construct each perpendicular bisector.

To release the tool press **[esc]**

Maths Tip

Perpendicular Bisector

This mathematical term contains two parts:

Perpendicular – Right angle   Bisector – Cut into two equal parts

To help keep the construction lines visually separate from the sides of the triangle, make the perpendicular bisectors 'dotted'.

Move the mouse over the perpendicular bisector, a screen tip appears: 'line'. Press **[Ctrl]** followed by **[menu]** and select **attributes**.

Use the up/down and left/right navigation to select the **dotted** line; repeat this process for the other two perpendicular bisectors.

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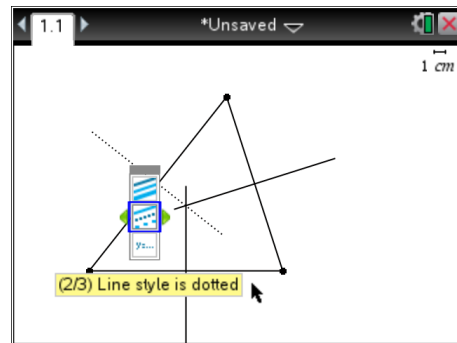
The intersection of the perpendicular bisectors can be marked by a point.

[Menu] > Points and Lines >

From the **points and line** menu select i**ntersection point(s)**. Select two of the perpendicular bisectors to create the point of intersection.
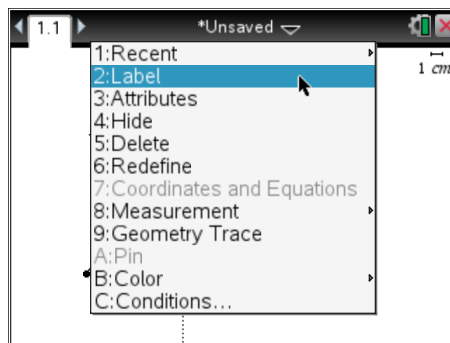
Repeat this for another pair of perpendicular bisectors.

Press **[esc]** to release the intersection point tool.
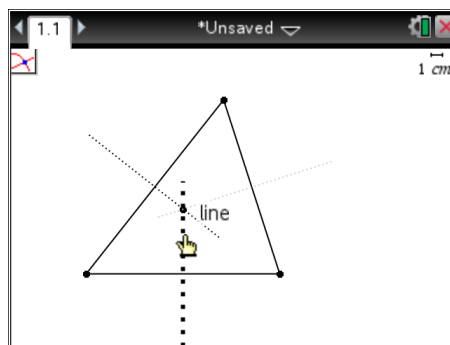
Move the mouse over the top of this new point and press **[Ctrl]** followed by **[menu]** to access the context menu.

Select **label** and mark the point of intersection: C.

Move the mouse over one of the triangle vertices. When it turns into a hand, hold the mouse down for a few moments to grab the point. Move the vertex of the triangle and watch point *C* as the triangle is changed.

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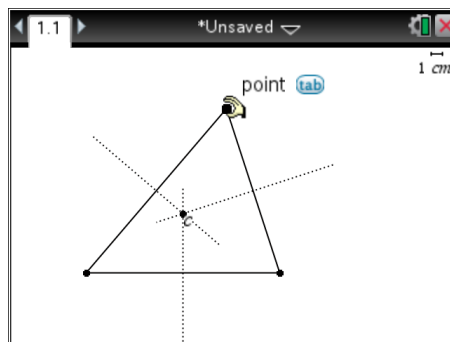
1. Do **all** three perpendicular bisectors meet at a single point?

    All three vertices appear to meet at a single point. (Observation)

2. How does the shape of the triangle affect the location of point **C**? (Comment on specific cases)

   There are three distinct situations for students to identify:

    Centre inside the triangle - Acute angled triangles.

    Centre outside the triangle – Obtuse angled triangles.

    Centre on side of the triangle – Right angled triangle.

    The first two situations should help formulate the third. If the triangle 'centre' is inside the triangle when all angles are less than 90 degrees and outside when one of the angles is greater than 90 degrees, then it seems logical that the centre will be on the boundary when the angle is 90 degrees.

    This third point for consideration can also be referenced in 'reverse'. (See above)

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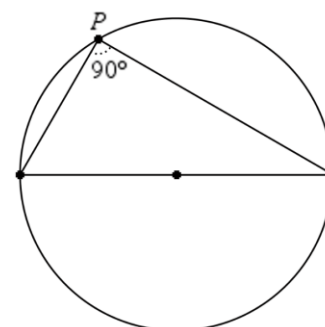
3.   Do you think point **C** represents the centre of the triangle? (Justify your answer)

*Answers will vary: Point C is one centre of the triangle. It 'appears to be' equidistant from each of the triangle's vertices. There are many other options for locating the 'centre' of a triangle.*

A circle can be drawn with the centre at point **C** and passing through one of the vertices of the triangle.
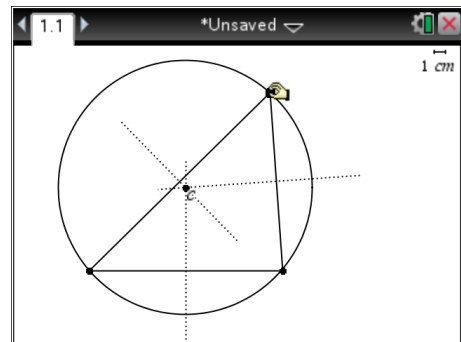
   [Menu] > Shapes > Circle

Click on point **C** followed by one of the triangle's vertices.

After the circle has been drawn, press **[esc]** to release the circle tool.
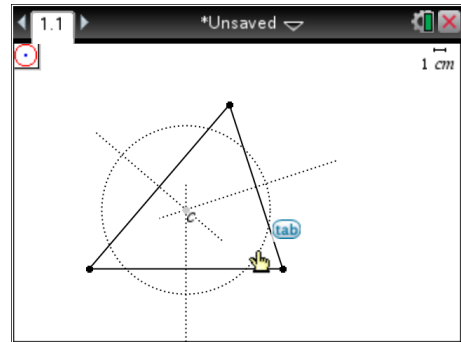
Drag the vertices of the triangle around the screen and observe how the circle changes.

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4.   What does the circle tell you about the distance from point **C** to the vertices of the triangle?

*The circle shows that the 'centre' is equidistant from each of the triangle's vertices.*
*Note: This is not a proof.*

Observation to Proof

"It is by logic that we prove, but by intuition that we discover" [Henri Poincaré 1908]. Manipulating the triangle and observing the circle provides a degree of intuition about what may be true. Proof provides irrefutable evidence that our observations will always hold true.

Use the **shapes** menu to draw another triangle. Place the vertices according to the image shown opposite:

(a) Vertex of the triangle

(b) Intersection of perpendicular bisector and side of triangle

(c) 'Centre' of triangle

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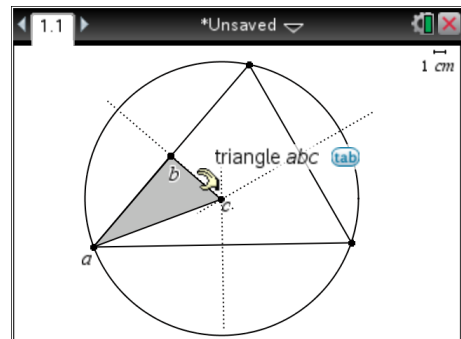
To make the triangle stand out it can be shaded (coloured).

Move the mouse over the top of this new triangle and press **[Ctrl]** followed by **[menu]** and select **Colour – Fill Colour**.
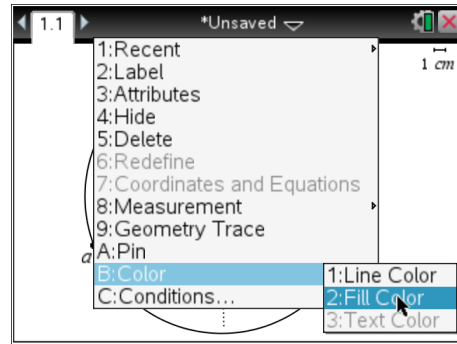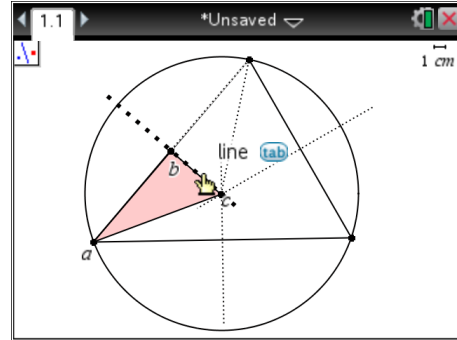
This triangle can be reflected.

[Menu] > Transformation > Reflection

Click on the new triangle (△abc) followed by the perpendicular bisector that defines one of the sides of the triangle.

>   (Shown opposite)

Colour in the reflected triangle.

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5. Drag the vertices of the original triangle and observe the movement of △ABC and its reflection. What do you notice about these two triangles?

    As △BCD is a reflection of △ABC then the two triangles must be congruent. Furthermore △BCD appears to have vertex D on the original triangle.

6. Use your knowledge of congruent triangles and reflections to prove that △ABC and △BCD (reflection) are congruent.

    - Side BC is common to △ABC and △BCD
    - Side AB = BD since BC is a bisector.
    - ∠ABC = ∠DBC (90° - perpendicular)

    Therefore by SAS △ABC is congruent with △BCD.
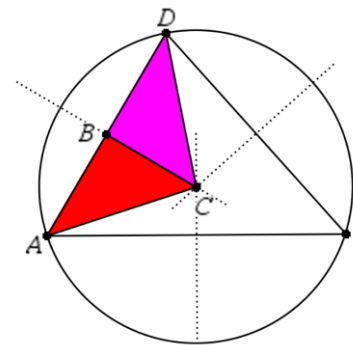
7. Use congruent triangles to prove that point **C** is equidistant from each of the original triangle's vertices.

    Since △BCD ≡ △ABC then it follows that AC = DC. Therefore points A and D are equidistant from the centre of the triangle. The same approach can be used using triangles formed around the other perpendicular bisectors therefore AC = EC where E is the third vertex on the original triangle, hence AC = DC = EC.

Teacher Notes:

Check out the other "Triangle Centre" activities on the Texas Instruments Australia website.

>   http://education.ti.com/australia   (Senior Curriculum Inspirations)

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