Which country is best?

Teachers Teaching with Technology™
Professional Development from Texas Instruments

TI-Nspire CXII/CXII CAS	Investigation	Teacher	50 min

Which country is best at the *World Games*?

At each *World Games*, the performance of each country is ranked by the total number of medals obtained. Is this the most suitable method for ranking country performance? In this task you will analyse the medals results over the last century and explore other possible ways of ranking performance, factoring in variables such as the type of medal, and the population and economic wealth of each country.

Working with the *World Game* medals database file

The *World medals database* (in the calculator file **worldgames.tns**) is a collection of medal records for each country that has been represented at the *World Games* over more than a century. As well as medals won, it provides some other basic data about country population, GDP per capita, and land area. Note that some of the data will refer to countries that no longer exist (e.g. Soviet Union), and occasionally some demographic information is based on the most recent estimates of their values (e.g. GDP per capita).

There are 9 statistical variables in this database, gathered from various sources. These variables contain data about each of the 215 countries listed.

- *country*   The name of the country/nation represented at each *World Games*
- *games*   The number of World Games at which each country was represented
- *gold*   The total number of gold medals won by each country
- *silver*   The total number of silver medals won by each country
- *bronze*   The total number of bronze medals won by each country
- *medals*   The total number of medals (of any type) won by each country
- *pop_mill*   The population of each country in millions, rounded to 2 d.p.
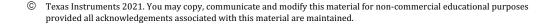- *area_km2*   The total area of each country in square kilometres
- *gdp_cap*   The gross domestic product (per capita) of each country

Here is a screen shot of some of the data – each **column** represents one of the nine statistical variables, and each **row** represents the collected records for each of the 215 listed countries.

[Note: the screens shown have been taken using the TI-Nspire CAS Teacher Software, which helps display the database – scroll through to view the records].

	A country	B games	C gold	D silver	E bronze	F medals	G pop_mill	H area_km2	I gdp_cap	J
1	Afghanis...	14	0	0	2	2	27.66	652230	549	
2	Albania	8	0	0	0	0	2.89	28748	5064	
3	Algeria	13	5	4	8	17	40.4	2381740	4363	
4	American..	8	0	0	0	0	0.06	199	11200	
5	Andorra	11	0	0	0	0	0.08	468	40403	
6	Angola	9	0	0	0	0	24.38	1246700	2891	
7	Antigua a..	10	0	0	0	0	0.09	443	12870	
8	Argentina..	24	21	25	28	74	43.59	2780400	8693	
9	Armenia	6	2	6	6	14	2.96	29743	4364	
10	Aruba	8	0	0	0	0	0.11	29	21833	
11	Australia	26	147	166	188	501	24.17	7741220	56307	
12	Austria	27	18	33	36	87	8.74	83871	47055	
13	Azerbaija..	6	7	11	24	42	9.76	86600	5593	
14	Bahamas..	16	6	2	6	14	0.38	13878	23971	
15	Bahrain	9	2	1	0	3	1.57	760	11011	

11

Author: D. Tynan

TEXAS INSTRUMENTS

Maintaining database integrity

The TI-Nspire **Lists and Spreadsheet App** is a powerful and flexible tool for analysing numerical data, and is particularly useful for column-wise and cell-wise analysis. However, the analysis of the *World Games* medals database requires 'row-wise' calculations (each country has its own row of data), particularly the ability to sort complete rows of the database by one or more of the column variables (eg medals). To maintain the integrity of the database, all the above 9 variables have been locked. When sorting is needed, the data from relevant variables are first copied to new variables, and then the sorting command is executed.

How to sort the *World Games Medals Database*

To sort a statistical variable (e.g. "sortvar") from the *World Games Medals Database* in descending order <u>by</u> a particular variable (e.g. "byvar"), the following command is used.

> **SortD byvar, sortvar**   (or **SortA** for an ascending sort)

This command can be used in conjunction with a couple of commands to store the relevant variables to achieve the required sorted results. The use of *sortvar* and *byvar* is necessary to avoid any potential changes to the row structure of the original database.

As an example, to sort and display the performance of each country by total number of gold medals won, the following four commands are needed:

1. **sortvar:=country**   [stores the values of the variable *country* into a variable *sortvar*]
2. **byvar:=gold**   [stores the values of the variable *gold* into a variable *byvar*]
3. **SortD byvar,sortvar**   [performs a descending sort of *sortvar* (*country*) by *byvar* (*gold*)]
4. **sortvar**   [display the values of sorted values of *sortvar*]

These commands can be typed in a single line, separated by the colon (":") character. The following screen shows the result of these commands in a single line.
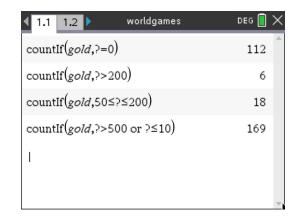
> *sortvar:=country*: *byvar:=gold*: SortD *byvar,sortvar*: *sortvar*
> {"United States","Soviet Union","Great Britain","China","France","Italy"▸

[Note: You will need to make use of similar commands to answer some of the questions below.]

How to count values in the *World Games Medals Database*

The TI-Nspire has a **countif** command that can be used to count the number of values that meet a particular condition [syntax is **countif(list, condition)**]. Here are some examples for the number of gold medals won (using the *gold* variable).

Note that the examples shown make use of the '**?**' character, which can be located on the calculator keyboard just below the **ENTER** key.

	worldgames	DEG
countIf($gold,? =0$)		112
countIf($gold,? >200$)		6
countIf($gold, 50 \leq ? \leq 200$)		18
countIf($gold,? >500$ or $? \leq 10$)		169

Author: D. Tynan

TEXAS INSTRUMENTS

Analysing the performance of countries

Use the **sortD**, **SortA** and/or the **countif** commands (as shown above) and the data from the *World Games Medals Database* to help answer the following questions.

[Note: variable names can be entered via the **VAR** key].

Question 1

List the top 10 ranked countries by **total** medals.

Rankings

1st	2nd	3rd	4th	5th
United States	Soviet Union	Great Britain	France	Germany
6th	7th	8th	9th	10th
Italy	China	Australia	Sweden	Hungary

$sortvar$:=$country$: $byvar$:=$medals$: SortD $byvar,sortvar$: $sortvar+byvar$

$\{$"United States"+2523,"Soviet Union"+1010,"Great Britain"+851,"France"+731,"Germany"+615,"Italy"+577,"China"+546,"Au▸

$sortvar$

$\{$"United States","Soviet Union","Great Britain","France","Germany","Italy","China","Australia","Sweden","Hungary","Japan"▸

Question 2

List the top 10 ranked countries by **gold** and **silver** medals.

Rankings

1st	2nd	3rd	4th	5th
United States	Soviet Union	Great Britain	France	China
6th	7th	8th	9th	10th
Germany	Italy	Hungary	Sweden	Australia

$sortvar$:=$country$: $byvar$:=$gold+silver$: SortD $byvar,sortvar$: $sortvar+byvar$

$\{$"United States"+1817,"Soviet Union"+714,"Great Britain"+558,"France"+462,"China"+391,"Germany"+385,"Italy"+384,"Hun▸

$sortvar$

$\{$"United States","Soviet Union","Great Britain","France","China","Germany","Italy","Hungary","Sweden","Australia","East G▸

Question 3

How many countries have won between 100 and 500 medals (inclusive) in total?

$countIf(medals,100\leq?\leq500)$ 25

Author: D. Tynan

TEXAS INSTRUMENTS

Question 4

How many countries have won more than 200 medals that were either gold or silver?

$\text{countIf}(gold+silver, ?>200)$ 13

Question 5

How many countries have never won a World Games medal?

$\text{countIf}(medals, ?=0)$ 72

Question 6

In official overall rankings used for the *World Games*, only total medals are considered. An alternative 'weighted' system of ranking is to award 3 points for a gold medal, 2 points for a silver medal, and 1 point for a bronze medal. The following command will produce the country rankings for this alternative ranking system.

$sortvar:=country: \quad byvar:=3\cdot gold+2\cdot silver+1\cdot bronze: \quad \text{SortD } byvar,sortvar: \quad sortvar$

How would this alternative ranking system affect Australia's overall ranking?

$sortvar:=country: \quad byvar:=3\cdot gold+2\cdot silver+1\cdot bronze: \quad \text{SortD } byvar,sortvar: \quad sortvar+byvar$
  {"United States"+5362,"Soviet Union"+2119,"Great Britain"+1672,"France"+1409,"Germany"+1191,"Italy"+1167,"China"+116▸

$sortvar$
  {"United States","Soviet Union","Great Britain","France","Germany","Italy","China","Hungary","Australia","Sweden","Japan"▸

Australia would drop from 8th to 9th if this alternative ranking system was adopted.

Question 7

Can you find another 'weighted' system of that would improve Australia's overall ranking?

Answers will vary, but if …

Points awarded for gold ≥ points awarded for silver ≥ points awarded for bronze

then the best ranking possible for appears to be 8th, which occurs if all medals attract the same number of points.

Author: D. Tynan

TEXAS INSTRUMENTS

Question 8

Another way of ranking the performance of countries is to consider the number of *World Games* in which each country is represented. List the top 10 ranked countries by using the ratio **medals/games**.

Rankings

1st	2nd	3rd	4th	5th
Soviet Union	United States	East Germany	Russia	China
6th	7th	8th	9th	10th
West Germany	Germany	Great Britain	France	Italy

Comment on any changes in the rankings from the official overall rankings.

$$sortvar := country: \quad byvar := \frac{medals}{games}: \quad SortD \; byvar, sortvar: \quad sortvar + byvar$$

$$\left\{ \text{"Soviet Union"} + \frac{1010}{9}, \text{"United States"} + \frac{841}{9}, \text{"East Germany"} + \frac{409}{5}, \text{"Russia"} + 71, \text{"China"} + \frac{273}{5}, \text{"West Germany"} \right.$$

sortvar

$$\left\{ \text{"Soviet Union"}, \text{"United States"}, \text{"East Germany"}, \text{"Russia"}, \text{"China"}, \text{"West Germany"}, \text{"Germany"}, \text{"Great Britain"} \right.$$

Answers will vary, but the Soviet Union has the highest ranked value of *medals* per *World Games*. Note also that Russia, East and West Germany are now in the top 10 countries by this measure.

Question 9

It has been suggested that the ranking performance of countries should also consider each of the following factors:

  a. the population of each country (for example using population in millions [*pop_mill*])

  b. the relative wealth of each country (for example by using the GDP per capita [*gdp_cap*])

  c. the geographic size of each country (for example the area in square kilometres [*area_km2*])

Use appropriate ratios to determine the effect of each of the above three factors to see how the top 5 ranked countries is affected.

a. Top 5 rankings using the ratio _____medals/pop_mill_____

1st	2nd	3rd	4th	5th
Finland	Hungary	Sweden	Bahamas	Denmark

$$sortvar := country: \quad byvar := \frac{medals}{pop\_mill}: \quad SortD \; byvar, sortvar: \quad sortvar + byvar$$

$$\left\{ \text{"Finland"} + 55.1913, \text{"Hungary"} + 50., \text{"Sweden"} + 49.8486, \text{"Bahamas"} + 36.8421, \text{"Denmark"} + 33.9161, \text{"Bulgaria"} + 30.4895, \text{"Norway"} \right.$$

sortvar

$$\left\{ \text{"Finland"}, \text{"Hungary"}, \text{"Sweden"}, \text{"Bahamas"}, \text{"Denmark"}, \text{"Bulgaria"}, \text{"Norway"}, \text{"Jamaica"}, \text{"Estonia"}, \text{"East Germany"}, \text{"New Zeala"} \right.$$

Author: D. Tynan

TEXAS INSTRUMENTS

b. Top 5 rankings using the ratio _____medals/gdp_cap_____

1st	2nd	3rd	4th	5th
Ethiopia	Kenya	China	United States	Ukraine

$$sortvar:=country:\ byvar:=\frac{medals}{gdp\_cap}:\ SortD\ byvar,sortvar:\ sortvar+byvar$$

{"Ethiopia"+0.086677,"Kenya"+0.085406,"China"+0.064961,"United States"+0.04694,"Ukraine"+0.038511,"Russia"+0.036142▸

sortvar

{"Ethiopia","Kenya","China","United States","Ukraine","Russia","Cuba","North Korea","Hungary","Estonia","Romania","Bulg▸

c. Top 5 rankings using the ratio _____medals/area_km2_____

1st	2nd	3rd	4th	5th
Bermuda	Jamaica	Singapore	Netherlands	Ivory Coast

$$sortvar:=country:\ byvar:=\frac{medals}{area\_km2}:\ SortD\ byvar,sortvar:\ sortvar+byvar$$

{"Bermuda"+0.018868,"Jamaica"+0.007097,"Singapore"+0.006954,"Netherlands"+0.00686,"Ivory Coast"+0.005825,"Grenada"▸

sortvar

{"Bermuda","Jamaica","Singapore","Netherlands","Ivory Coast","Grenada","Hungary","Belgium","Switzerland","Denmark","E▸

Comment on any changes in the rankings from the official overall rankings due to these 3 factors.

Possible comments

- Considering these factors (or ratios), the top 5 rankings are quite different

- Nordic countries are well represented if population is considered as a factor

- Two African countries feature is country wealth (by GDP per capita) is considered as a factor

- Small countries are well represented if country area is considered as a factor.

Question 10

Multiple factors can be considered. For instance, the ratio (*medals/games*)/*gdp_cap* considers the impact of the average number of medals per *World Games*, as well as the relative wealth of each country. Which of the countries in this 'top 10" are also in the 'official top 10' (total medals)?

$$sortvar:=country:\ byvar:=\frac{\frac{medals}{games}}{gdp\_cap}:\ SortD\ byvar,sortvar:\ sortvar+byvar$$

{"Ethiopia"+$\frac{54}{8099}$,"China"+$\frac{273}{42025}$,"Ukraine"+$\frac{5}{779}$,"Kenya"+$\frac{103}{16884}$,"Russia"+$\frac{71}{11787}$,"North Korea"+$\frac{27}{8500}$,"E▸

sortvar

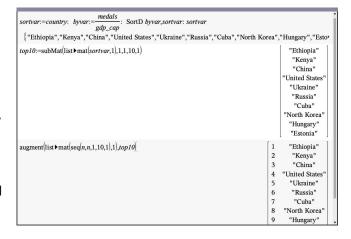{"Ethiopia","China","Ukraine","Kenya","Russia","North Korea","Estonia","Uzbekistan","Belarus","United Stat▸

Only United States and China are in both top 10 lists.

Author: D. Tynan

TEXAS INSTRUMENTS

Teacher notes

- This task is intended to be attempted after students have completed at least some work on Year 9 or Year 10 statistics, or even for Year 11 General Maths depending on the cohort of students.

- The task also focusses on the concept of a statistical variable, and in constructing sums and quotients of those variables to aid in the analysis of a large data set.

- This activity is designed for students as an investigation of the *World Games Medals Database*, and requires the teacher to transfer the file "**worldgames.tns**" calculator file to each of your student's calculators. Students will need to be shown how to use **sort** and **countif** commands to answer the questions. It helps if the teacher has access to an IWB or data projector so that students may easily view the calculator screen (i.e. via TI-Nspire CAS software). There is a student worksheet in which students can record their answers.

- A side benefit of this task is that it demonstrates to students how a limited set of commands can be combined from the calculator page to mimic common programming ideas (eg storing subsets of a dataset as temporary variables, combining commands in a single line)

- There are other analyses possible for this data, for example examining properties of each of the numerical variables separately and managing & visualizing large data sets where the range of values is great.

- The database was compiled from various sources, and it took a lot of time to compile – there are many footnotes that could have been added to do with explanations about entries about GDP per capita, changes in political geography over the last century, and countries that no longer exist. Every attempt has been made to ensure that the data is accurate, however if you find an error, please feel free to update the database!

- The display of results to database queries is limited by the screen size and resolution of the calculator. On the screenshot right are some optional commands that can be used to improve that aspect. They are not critical to the task – they just make it easier to view results on the calculator.

- The task and the database work equally well on the TI-Nspire CXII and CXII CAS. The only exception is the following.
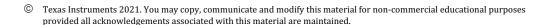
  - On the CXII CAS, you can use the following command to sort and display both sorted variables

    $sortvar := country$: $byvar := gold$: $\text{SortD } byvar, sortvar$: $sortvar + byvar$

  - On the CXII, you can use the following command to sort and display *sortvar* (display *byvar* in following command)

    $sortvar := country$: $byvar := gold$: $\text{SortD } byvar, sortvar$: $sortvar$

Author: D. Tynan

TEXAS INSTRUMENTS