Vertical and Phase Shifts

TIMATH.COM: PRECALCULUS

TEACHER NOTES

ID: 9608

Activity Overview

Students explore vertical and phase shifts of sine and cosine functions, after a brief review of period and amplitude. Students will use values in lists to change the values of parameters in trigonometric functions; they will determine the effect that each change has upon the shape of the graph. They will then use this knowledge to write a sine function as a cosine function.

Topic: Trigonometric Functions

- Analyze and predict the effects of changes in A, B and C on the graphs of A sin(Bx + C), A cos(Bx + C), and A tan(Bx + C) and interpret A, B, and C in terms of amplitude, period, and phase shifts.
- Approximate the amplitude, frequency, and phase shift of the primary trigonometric functions by graphing.

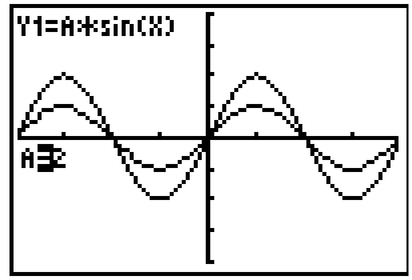
Teacher Preparation and Notes

- Students should already be familiar with the graphs of the sine and cosine functions, and they should also have some experience in determining the period (in radians) and identifying the amplitude of a trigonometric function.
- The graphs of trigonometric functions that students will view on their calculators have been constructed such that the horizontal scale is in multiples of $\frac{\pi}{2}$.
- **To download the student worksheet, go to education.ti.com/exchange and enter "9608" in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Can You Make My Graph? (TI-Nspire technology) — 9161
- Sinusoidal Transformations (TI-Nspire technology) — 10084
- Numb3rs – Season 1 – "Counterfeit Reality – Changing Sines (TI-84 Plus family) — 7716



This activity includes screen captures taken from the TI-84 Plus Silver Edition. It is also appropriate for use with the TI-83 Plus and TI-84 Plus but slight variances may be found within the directions.

Compatible Devices:
- TI-84 Plus Family

Software:
- Transformation Graphing

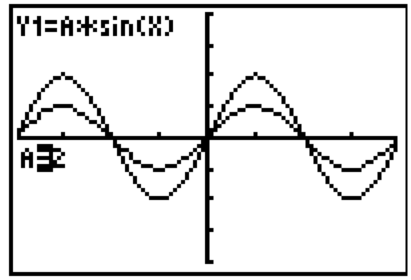
Associated Materials:
- VerticalAndPhaseShifts_Student.pdf
- VerticalAndPhaseShifts_Student.doc
- PHSESHFT.8xp
- TRIGCOMB.8xp

Click HERE for Graphing Calculator Tutorials.

Before beginning the activity, students should clear out any functions from the [Y=] screen, turn off all Stat Plots, and make sure that the calculator is in Radian mode. They will also utilize the **Transformation Graphing App**. To start this app, press the [APPS] and select **Transfrm** from the list and press [ENTER] twice to activate the application.

Problem 1 – Amplitude

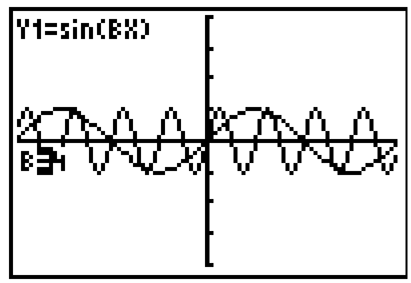
This problem allows students to review the amplitude of a function of the form $f(x) = a \sin(x)$. By changing the value of ***A***, students should conclude that the sine curve is vertically stretched by a factor of $|a|$. Thus, amplitude = $|a|$. Be sure to ask what effect the sign of a has on the graph (if a is negative, then the curve is reflected over the x-axis).



Problem 2 – Period

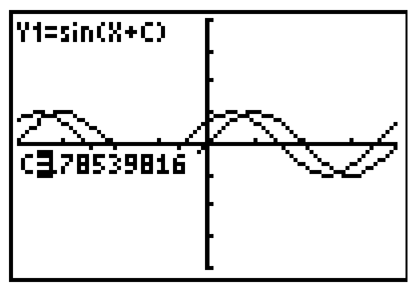
This problem allows students to review the period of a function of the form $f(x) = \sin(bx)$. By changing the value of ***B***, students will find that the value of b affects the horizontal stretch of this function and thus changes the period of the function.

After some examination, students should be able to identify the relationship: period = $\frac{2\pi}{b}$.



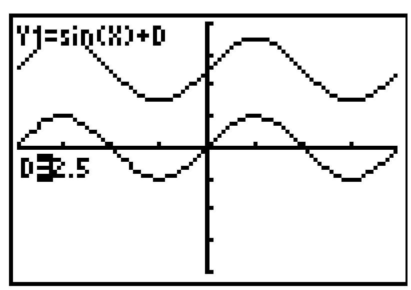
Problem 3 – A Simple Phase Shift

The graph of $f(x) = \sin(x + c)$ is explored next. Students might predict that a change in c will result in a horizontal shift a certain number of units (here called a phase shift), but how that number of units relates to c will not be immediately clear. This is explored in Problem 5.



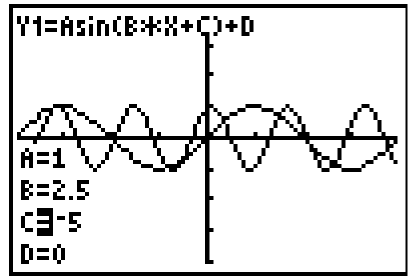
Problem 4 – Vertical Shift

The parameter d in the graph of $f(x) = \sin(x) + d$ is now considered. Before students change the value of ***D***, ask them to predict what will happen by first considering a different function, $y = x^2$. Ask them how to obtain the graph of $y = x^2 + 3$ from the graph of $y = x^2$ (translate the graph up three units). Students should confirm that the vertical shift is equal to this parameter; that is, vertical shift = d.



Problem 5 – Combining Transformations

While students may be wondering why they cannot declare "phase shift = –*c*," have them again consider the graph of $y = x^2$. Ask them how to obtain from it the graph of $y = (x + 3)^2$ (translate three units to the left). Then display the equations $y = [4(x + 3)]^2$ and $y = (4x + 3)^2$, and elicit from students that only the former represents a shift three units to the left. At this point students should realize that the phase shift of $f(x) = a \sin(bx + c) + d$ depends on two parameters: *b* and *c*.



To establish exactly how *b* and *c* determine the phase shift, students may change the values of the variables. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the phase shift when *b* = 1 and *c* = 2, when *b* = 2 and *c* = 1, and when *b* = 2.5 and *c* = –5. (It is –2, –0.5, and 2, respectively.) Examining these values, students should conclude that phase shift = $-\frac{c}{b}$.

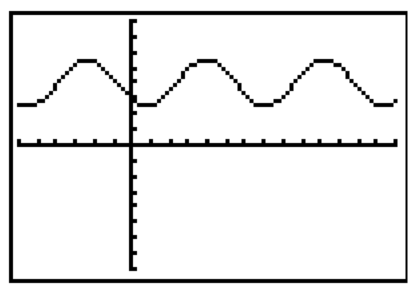
Note: It is easiest to identify this relationship if parameters *a* and *d* are left as initially set (*a* = 1 and *d* = 0). After the relationship is determined, changing the other values can verify that neither *a* nor *d* affects the phase shift.

Problem 6 – Bringing It All Together

Students are asked to summarize their findings from Problems 1–5. Make sure students have correctly completed these answers before proceeding further.

Next, students are asked to return to Problems 1–5 and replace each "sin" in the function definition to "cos" to verify that the same characteristics hold true.

Finally, students will apply what they have learned about vertical and phase shifts. They are given the equations and graphs of two sine functions and asked to find equations of cosine functions that coincide. Students should observe that the values of *a*, *b*, and *d* remain the same for each sine/cosine pair; the only difference occurs in the value of *c*. Because these functions are periodic, there are infinitely many equations that satisfy each condition. Be sure to check students' equations.



Solutions

Problem 1

- students should conclude that the sine curve is vertically stretched by a factor of |*a*|
- if *a* is negative, then the curve is reflected over the *x*-axis
- amplitude = |*a*|

Problem 2

Students will find that the value of b affects the horizontal stretch of this function and thus changes the period of the function. After some examination, students should be able to identify the relationship: period = $\frac{2\pi}{b}$.

Problem 3

Students might predict that a change in c will result in a horizontal shift a certain number of units (here called a phase shift), but how that number of units relates to c will not be immediately clear.

Problem 4

Students should confirm that the vertical shift is equal to this parameter. (vertical shift = d).

Problem 5

While students may be wondering why they cannot declare "phase shift = $-c$," have them again consider the graph of $y = x^2$. Ask them how to obtain from it the graph of $y = (x + 3)^2$ (translate 3 units to the left). Then display the equations $y = [4(x + 3)]^2$ and $y = (4x + 3)^2$, and elicit from students that only the former represents a shift 3 units to the left. Students should realize that the phase shift of $f(x) = a \sin(bx + c) + d$ depends on <u>two</u> parameters: b and c.

To establish exactly how b and c determine the phase shift, students may are to change the values. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the phase shift when $b = 1$ and $c = 2$, when $b = 2$ and $c = 1$, and when $b = 2.5$ and $c = -5$. (It is –2, –0.5, and 2, respectively.) Examining these values, students should conclude that phase shift = $-\frac{c}{b}$.

<u>**Note**</u>**:** It is easiest to identify this relationship if parameters a and d are left as initially set ($a = 1$ and $d = 0$). After the relationship is determined, dragging the sliders can verify that neither a nor d affects the phase shift.

Problem 6

- amplitude = $|a|$, period = $\frac{2\pi}{b}$, phase shift = $-\frac{c}{b}$, vertical shift = d
- $f(x) = 1.5 \cos\left(x + \frac{3\pi}{4}\right) + 4$
- $g(x) = 3 \cos\left(2x - \frac{\pi}{2}\right) - 5$

Students should observe that the values of a, b, and d remain the same for each sine/cosine pair; the only difference occurs in the value of c. Since these functions are periodic, there are infinitely many that satisfy each condition. Be sure to check students' equations.