Math Objectives

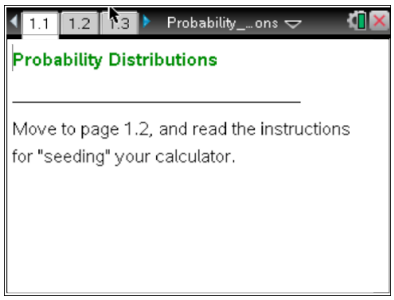
- Students will compare the distribution of a discrete sample space to distributions of randomly selected outcomes from that sample space.
- Students will identify the structure that emerges as more and more data are added to the distribution (that is, the features of the distribution of random processes stabilize as the sample gets larger).
- Students will recognize that relative frequencies of particular outcomes after few observations are more likely to deviate substantially from expected values than are relative frequencies based on more observations.
- Students will use appropriate tools strategically (CCSS Mathematical Practices).
- Students will look for and make use of structure (CCSS Mathematical Practices).

Vocabulary

- distribution
- variability
- sample space
- outcome
- relative frequency

About the Lesson

- This lesson involves generating single outcomes from a given sample space and observing how the distribution shape develops as the number of observations increases.
- As a result, students will:
  - Compare with their classmates' distributions to note the large variability that can occur in relative frequency measures with only a few observations.
  - Observe the generation of a very large number of outcomes to visualize the law of large numbers.
  - Observe how the probability of a specific outcome stabilizes as the number of observations increases.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Use the RandSeed command to see the random number generator.

Lesson Materials:

Student Activity

- Probability_Distributions_Student.pdf
- Probability_Distributions_Student.doc

TI-Nspire document

- Probability_Distributions.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

TI-Nspire™ Navigator™ System

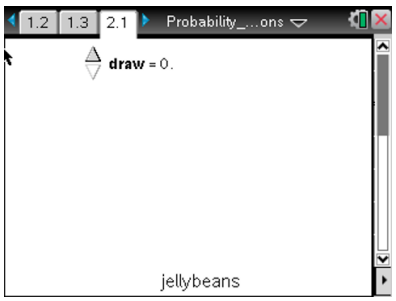
- Send the .tns file to students.
- Use Screen Capture to display multiple distributions.
- Use Quick Poll to compare student results.

Discussion Points and Possible Answers

Teacher Tip: Prior to beginning this activity, each student should seed the random number generator by reading the instructions on 1.2 and then moving to the Calculator page on 1.3 to execute them. The entire command can be typed directly, using the alpha keys on the keyboard.

Teacher Tip: The activity assumes that drawing jellybeans is done with replacement, where after drawing a jellybean and recording its color, it is replaced in the bag, which is then thoroughly mixed and a new jellybean is drawn.

Move to page 2.1.



1. Each click of the arrow on page 2.1 will draw a jellybean from a bag of jellybeans and plot the jellybeans according to color.
   a. Draw until you have ten jellybeans. How many different jellybean colors do you think are in the bag? Explain your reasoning.

   Answer: Some students may say they do not have enough information yet. Others might respond, "I have about the same number of each color."; "I have four colors and mostly yellow (six yellow jelly beans)."

   b. Predict the number of each jellybean color you will have if you generate 10 more jellybeans and add them to the distribution.

   Sample Answers: Student responses should be based on the distribution they have already generated, indicating “more of the same,” so the predictions might be double their results from the first ten draws.

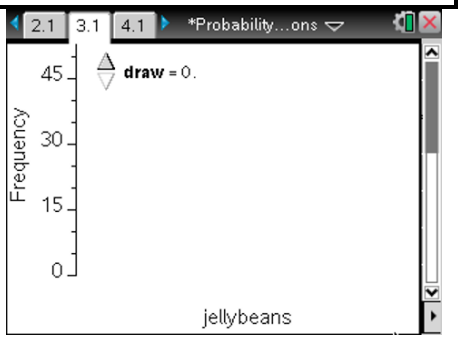
c. Draw 10 more jellybeans. Were you surprised by the resulting distribution? Explain why or why not.

Sample Answers: Most students should be surprised because with only 10 jellybeans, the distribution of results will often be very different from their former distribution. For example, a student might have 0, 1, 2, 5, 1 as the distribution of the first ten draws and 0, 1, 2, 11, 6 when the next ten draws are added.

TI-Nspire Navigator Opportunity: ***Screen Capture.***
See Note 1 at the end of this lesson.

Move to page 3.1

The arrow will count the jellybeans in sets of five and show a bar graph of the number of jellybean colors as sets of five are added to the total. (Note that moving the cursor over each bar provides information about the number of jellybeans represented by the bar.)



2. Click the arrow until you have generated a distribution that has 50 jellybeans.
   a. Make a rough sketch of the distribution you generated and describe its characteristics.

Sample Answers: Individual distributions will be somewhat different.

Teacher Tip: The rough sketch will enable students to start identifying the major features of the distribution. Encourage them to try to highlight the striking features—the maximums and the general shape—rather than trying to be exact.

Note that the graph is a frequency distribution. The graphs on page 4.1 will be relative frequency distributions. Be sure students understand the difference between the two ways of keeping track of the numbers of each color.

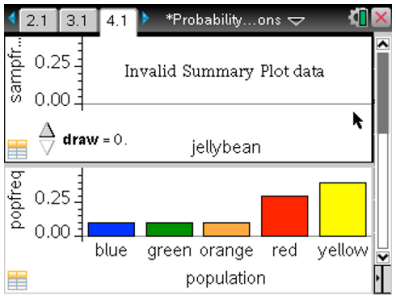
   b. What is the percentage of each jellybeans color in your distribution? Which jellybean color seems to be drawn most often?

Sample Answers: Answers will vary but will probably indicate that getting a red or yellow jellybean has the greatest chance of occurring. The percentages of each color might be 8%, 17%, 9%, 30%, and 36%.

c. Continue clicking the arrow to generate a distribution that has 100 jellybeans. Sketch the distribution, and describe how it changed from your sketch in part a.

Sample Answers: Answers will vary, but plots across the room should become more and more similar.

Move to page 4.1.



The set of all possible outcomes from a chance experiment is called the **sample space** for that experiment. Here, the sample space contains the colors blue, green, orange, red, and yellow, but they are not equally likely to occur, as was evident from Questions 2 and 3.

3. The bottom graph represents the distribution of the number of jellybeans of each color in the bag.
   a. If you draw a jellybean from the bag, what is the probability of getting each jellybean color?

      Answer: $P(\text{blue}) = P(\text{green}) = P(\text{orange}) = 0.1$, $P(\text{red}) = 0.3$, $P(\text{yellow}) = 0.4$

   b. How does the distribution you generated and sketched in Question 3, part c compare to the actual population distribution of colors?

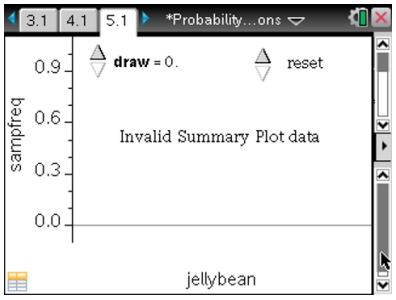
      Sample Answer: It looks about the same, with only very slight differences.

   c. Click the arrow to generate a distribution in the upper plot that has 50 sample elements. Describe what is happening to the distribution of the colors as the number of jellybeans increases.

      Sample Answer: The plot at the top becomes more and more like the one on the bottom.

TI-Nspire Navigator Opportunity: *Screen Capture.*
See Note 2 at the end of this lesson.

Move to page 5.1.



4. a. Draw 10 jellybeans and enter the observed relative frequency of getting each jellybean color in the first row of the table. (Note: Each trial consists of exactly 10 jellybeans.) Use the reset arrow to clear that set of ten jellybeans and draw a new set of ten jellybeans. Record the relative frequency of each color in the table. Repeat this process to fill in all five rows of the table.

Outcome/Run	blue	green	orange	red	yellow
First Trial	**0.1**	**0**	**0.3**	**0.3**	**0.3**
Second Trial	**0.2**	**0.1**	**0.2**	**0.2**	**0.3**
Third Trial	**0.3**	**0**	**0.2**	**0.3**	**0.2**
Fourth Trial	**0**	**0.2**	**0.2**	**0.5**	**0.1**
Fifth Trial	**0**	**0.1**	**0.3**	**0.2**	**0.4**

b. Based on your results in the table, do you think this is the same bag of jellybeans used on page 4.1? Why or why not?

Sample Answer: Students should notice that the outcomes for each color vary widely in the five trials and make it impossible to tell whether the jellybeans come from the same bag of jellybeans. For example, only the last trial had yellow as the most frequent color, which would suggest the jellybeans are from a new bag. But most of the time the jellybeans were not blue, green and orange, which is typical of the bag on the previous page. Some may decide to accumulate all of each color, getting for example 6 out of 50 blue (12%), 4 out of 50 green (8%), 12 out of 50 orange (24%), 15 out of 50 red (30%) and 13 out of 50 yellow (26%). The percentages are close to the previous bag for blue, green and red but way off for yellow and orange. There is just not sufficient evidence to accept the fact that this is a new bag.

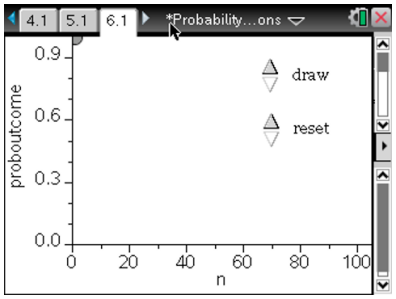
Teacher Tip: Be sure students recognize that a relative frequency of blue jellybeans on the first draw of 0.1 and of 0.2 on the second means that three jellybeans out of 20 were blue.

Teacher Tip: It is important to share student responses to question 2b. Some of them may find that the results in 10 sample outcomes give a fair indication of the expected probability for each outcome, but overall there should be a large amount of variability. Students should recognize from this and their work in Questions 2–4 that smaller samples are more likely to deviate substantially from expected values than are larger ones.

Teacher Tip: Question 5b brings up the question, 'how much evidence do you need to believe something has changed?' This can be an informal introduction to hypothesis testing and deciding whether the evidence supports rejecting or failing to reject the null hypothesis.

Teacher Tip: This is a good time to talk about "hot hands" in basketball or similar streaks in sports where the media make headlines by claiming that someone is "due" for a hit.

Move to page 6.1.



5. Draw four jellybeans.
   a. Make a sketch of the plot, describe what occurs, and interpret its meaning.

      Sample Answers: Student outcomes will vary, but the first element in each should be either a 0 or a 1, and the four dots should show the progress of the changing relative frequency of successes.

   b. Interpret the results in terms of the probability of obtaining that jellybean color after each sample.

      Answer: The "*y*-coordinate" of each point gives the "current" relative frequency of "marble is a 4" for the selections thus far.

   c. Continue to draw more jellybeans. If the jellybeans are drawn from the original bag used in questions 1 to 4, what jellybean color do you think you are counting? Explain how you know.

      Answer: The probability of the outcome varies a lot at the beginning but begins to level off and stabilize at 0.3, or 30%, after about 50 samples. It would probably be the red jellybeans.

TI-Nspire Navigator Opportunity: ***Quick Poll.***
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to:

- Identify the set of possible outcomes together and estimate the probabilities of those outcomes from repeating an experiment a large number of times.
- Understand the law of large numbers as a fundamental concept in statistics and that probability describes how the average relative frequency of an event is likely to be close to the actual probability of that event as the number of trials gets larger and larger.
- Recognize that relative frequencies of particular outcomes after few observations are more likely to deviate substantially from expected values than are relative frequencies based on more observations.

Assessment

Decide whether each of the following is always true, sometimes true, or never true. Explain your reasoning in each case.

1. A sample of 10 outcomes from a sample space will give you a good estimate of the probability of a specific outcome.

   Answer: Answers will vary, but “sometimes true” is the strongest statement that could be defended.

2. Over the long run, the probability of an outcome based on repeated sampling will converge to the expected value for that outcome.

   Answer: Always true (if “long run” is long enough)

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Note 1

Questions 1c, *Screen Capture*

This is a good time to use Screen Capture to collect student results and show the different distributions to the class.

Note 2

Question 2c, *Screen Capture*

This is another good chance for a Screen Capture to ensure students are performing the assignment correctly. Scroll through the captured screens so students can see the different possible distributions.

Note 3

Assessment, *Quick Poll*

Quick Poll can be used to assess student understanding using the two questions in the Assessment. Students should be prepared to explain their reasoning.