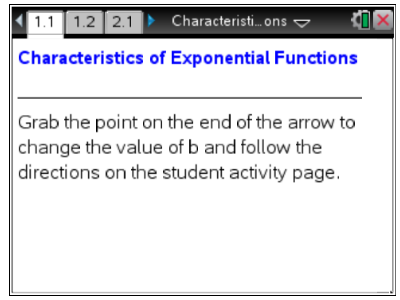
Characteristics of Exponential Functions



Math Objectives

- Students will identify the characteristics of exponential functions of the form $f(x) = b^x$, where $b > 1$.
- Students will identify the characteristics of exponential functions of the form $f(x) = b^x$, where $0 < b < 1$.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

- exponential function

About the Lesson

- This lesson involves investigating how the graph of an exponential function changes when $0 < b < 1$, $b = 1$, or $b > 1$.
- As a result, students will:
  - Graph an exponential function.
  - Describe the domain, range, and y-intercept of the exponential function.

TI-Nspire™ Navigator™

- Use Class Capture to examine patterns that emerge.
- Use TI-Nspire™ Navigator™ Teacher Software to review student documents.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Characteristics_of_Exponential_Functions_Student.pdf
- Characteristics_of_Exponential_Functions_Student.doc

TI-Nspire document

- Characteristics_of_Exponential_Functions_Student.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text.* Then press ctrl ☝ to grab the point and close the hand (☝).

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 1 at the end of this lesson.

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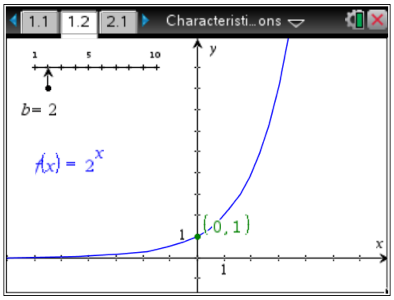

1. a. Describe some characteristics of the graph **f**(x) = 2^x, including the domain and range.

   Answer: The graph of $f(x) = 2^x$ curves upward. The graph of $f(x) = 2^x$ increases from left to right. The graph is close to the x-axis in Quadrant II. The y-intercept is (0, 1). The domain is the set of all real numbers, and the range is the set of all real numbers greater than 0.

   Teacher Tip: Since the graph of the function $f(x) = b^x$ behaves very differently in Quadrants I and II, you might want to briefly address the location of each quadrant. This will allow students to be more specific in their observations.

   b. Grab and move the point to increase the value of b. What happens to the graph as b increases? Do any of the characteristics you described stay the same? What changes?

   Answer: As b increases, the y-intercept remains (0, 1). The graph is still close to the x-axis in Quadrant II. The domain and range remain the same. As b increases, the graph gets steeper.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

2. a. Why do the graphs of $f(x) = 3^x$ and $f(x) = 5^x$ both pass through the point (0, 1)?

Answer: The graphs of $f(x) = 3^x$ and $f(x) = 5^x$ both pass through the point (0, 1) because the value of both expressions is 1 when 0 is substituted for x. For both functions, an input of 0 results in an output of 1: $3^0 = 1$ and $5^0 = 1$.

Teacher Tip: If students have no previous experience or are unsure about this exponent rule (and many are), discuss the rule. If students struggle to remember that $b^0 = 1$ for any value of b except 0 (that is, any non-zero number to the 0 power is 1), you might need to review some reasons for why this definition makes sense. For example, generate a pattern working from known powers of 2.

$2^3 = 8$ $\quad$ $2^2 = 4$ $\quad$ $2^1 = 2$ $\quad$ $2^0 = ?$

Note that the output is reduced by one half each time, so it seems logical 2^0 equals ½ of 2 or 1. This argument can be continued, if needed, to generate the definition for a negative exponent.

$2^{-1} = ½$ $\quad$ $2^{-2} = ¼$

You might need to repeat the pattern with different bases to be sure students understand this point. Stress that this is a definition, as is the definition of a negative exponent, and one of the important things about good definitions is that they should be consistent with other things we know.

b. Would it ever be possible to have a graph of the form $f(x) = b^x$ that does not pass through the point (0, 1)? Why or why not?

Answer: No, any non-zero base raised to the 0 power is 1.

3. Why is the graph of $f(x) = b^x$ a horizontal line when $b = 1$? Justify.

Answer: When $b = 1$, the graph of $f(x) = b^x$ is a horizontal line because repeated multiplication of 1 will always result in a product of 1. The function $f(x) = 1^x$ is the same as the function $f(x) = 1$. When any value of x is substituted into the function $f(x) = b^x$ where $b = 1$, the value will always be 1. This produces the set of points that include {… (–2, 1), (–1, 1), (0, 1), (1, 1), (2, 1), ...} and generates a horizontal line.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of this lesson.

4. Predict what will happen to the graph of $f(x) = b^x$ when the value of b is between 0 and 1 ($0 < b < 1$).

Answer: Since the product of two fractions whose value is between 0 and 1 is a smaller fraction than either of the two factors, the graph of $f(x) = b^x$ will decrease instead of increase. Therefore, as x increases, the values of $f(x)$ will decrease, and the graph will curve down instead of up.

> **Teacher Tip:** To assist students with their predictions, you can encourage them to choose specific b-values between 0 and 1 and square them. For example, $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$, $\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{9}$, and $\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{16}$. Since the products of these fractions are decreasing, the graph of any function $f(x)$ with a fraction whose value is between 0 and 1 raised to a power will decrease as the value of x increases.

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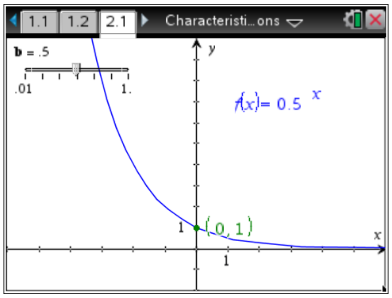


5. a. Test your prediction from question 4. Describe the characteristics of the graph of $f(x) = b^x$ when b is between 0 and 1.

Answer: When b is between 0 and 1, the graph of $f(x) = b^x$ curves down as the values of x increase. A fraction between 0 and 1 raised to a power decreases as the value of the exponent increases.

b. Explain any differences between this graph and the graph of $f(x) = b^x$ when b is greater than 1.

Answer: The graph is now decreasing from left to right instead of increasing because of the reason given above. The y-intercept remains (0, 1) because any non-zero number (fraction or integer) raised to the 0 power is 1. Both graphs are above the horizontal axis. In other words, the function values of both are always greater than 0.

> **Teacher Tip:** In order to place exponential functions within the context of other functions, you can briefly identify the characteristics of linear and quadratic functions, or you can refer to the activity *Comparing Linear and Exponential Functions*. For instance, linear functions can increase or decrease, but they all maintain the same shape: a line. Quadratic functions can open upward or downward, but they all maintain the same

general *u* shape: a parabola.
Exponential functions in the form $f(x) = b^x$, where $b > 0$, will either be high in the second quadrant and decrease approaching a horizontal line in the first quadrant or will be very close to a horizontal line in the second quadrant and increase rapidly in the first quadrant.

6. a. Eric noticed that the graph of $f(x) = b^x$ increases when b is greater than 1 ($b > 1$), and the graph of $f(x) = b^x$ decreases when b is between 0 and 1 ($0 < b < 1$). How could he mathematically justify this?

Answer: When $b > 1$, the graph of $f(x) = b^x$ increases because repeated multiplication of a number greater than 1 by a number greater than 1 always results in an increased product. When $0 < b < 1$, the graph decreases because repeated multiplication of a number between 0 and 1 by a number between 0 and 1 will always result in a decreased product. When $x = 1$, the value of the function will always be exactly 1.

Teacher Tip: You can have students choose a specific *b*-value that is greater than 1, multiply it repeatedly, and notice that the product increases. Similarly, students might choose a specific *b*-value that is between 0 and 1, multiply it repeatedly, and note that the product decreases.

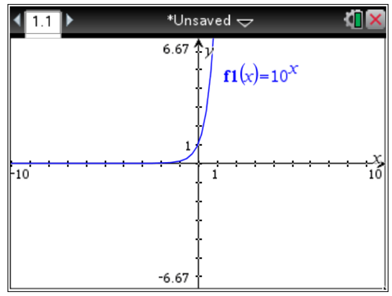
b. Cheryl wondered when $f(x) = b^x$ would equal 0. Use the TI-Nspire document on your handheld to investigate. What would you say to Cheryl?

Answer: The function $f(x) = b^x$ will never equal to 0. It is impossible for b^x to equal 0 as there is no way to obtain a 0 as a result of raising b to a power without $b = 0$. The graph gets closer and closer to the horizontal axis but will never actually reach it. The range for an exponential function in the form $f(x) = b^x$, where $b > 0$ (but $b \neq 1$), is all numbers greater than 0.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 4 at the end of this lesson.

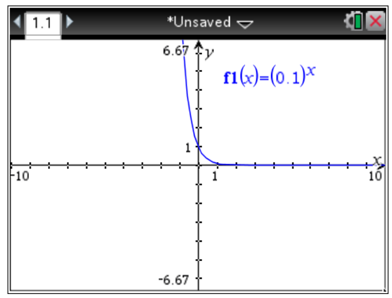
7. For each function below, sketch the graph. Identify the domain, range, *y*-intercept, and at least one other point on the graph.

a. $f(x) = 10^x$



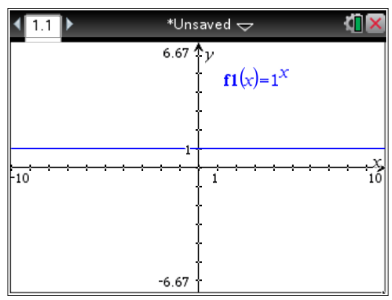
Answer: Domain: All real numbers;
Range: $\{f(x) \mid f(x) > 0\}$;
y-intercept: (0, 1);
another point: (1, 10)

b. $f(x) = (0.1)^x$



Answer: Domain: All real numbers;
Range: $\{f(x) \mid f(x) > 0\}$;
y-intercept: (0, 1);
another point: (–1, 10)

c. $f(x) = 1^x$



Answer: Domain: All real numbers;
Range: $\{f(x) \mid f(x)=1\}$;
y-intercept: (0, 1);
another point: (1, 1)

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- How to sketch a graph of a function of the form $f(x) = b^x$ for *b* greater than 0, including $0 < b < 1$, $b = 1$, and $b > 1$.
- For any exponential function, $f(x) = b^x$ where *b* is > 0, $f(x)$ will always be positive.

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Note 1

Class Capture and Live Presenter

Use Class Capture to make sure students are not having difficulty using the TI-Nspire document. Another approach is making the teacher or a student the Live Presenter to demonstrate to the class how to use the document.

As students are moving the point to increase the value of b, encourage them to discuss what is happening to the graph.

If students have difficulty, you can review the concepts from the lesson *Domain and Range of Exponential Functions*. It might be useful to pair students, assigning students of different abilities to the same pair.

Note 2

Question 1, Class Capture

Before the next question, assign half of the class to graph $f(x) = 3^x$ and the other half of the class to graph $f(x) = 5^x$. Use Class Capture to compare the graphs before answering the next question. Ask students for similarities and differences between the two graphs. Maybe record on the board or have students add a Notes page and take notes.

Note 3

Question 3, Class Capture

Before the next question, assign half of the class to graph two different functions that have a base between 0 and 1. Use Class Capture to compare the graphs before answering the next question. Ask students for similarities and differences between the two graphs. Record their responses on the board, or have students add a Notes page and take notes.

Note 4

Question 7, Quick Poll (Open Response)

For question 7, consider using a Quick Poll (open response) to get some of the students' answers.