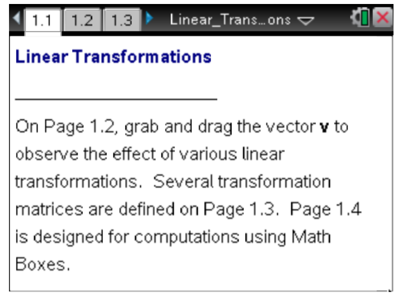
Linear Transformations
Student Activity

Name ______________
Class ______________

Open the TI-Nspire document *Linear_Transformations.tns.*



In this activity, you will visualize linear transformations from $\mathbf{R}^2$ to $\mathbf{R}^2$ to characterize some special cases. You will also observe the effect of a linear transformation in two dimensions, use your own words to describe the relationship between the input and output vectors, and use specific numerical results to support your conclusions.

A linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ can be represented by a matrix.
If T is a linear transformation that maps $\mathbf{R}^2$ to $\mathbf{R}^2$ and **v** is a 2×1 column vector, then the linear transformation can be written as $T(\mathbf{v})=\mathbf{m}\cdot\mathbf{v}$ for some 2×2 matrix $\mathbf{m}$.

The matrix $\mathbf{m}$ is called the transformation matrix.

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

The left work area is a Notes page with two interactive Math Boxes.

- In the first Math Box, define the matrix $\mathbf{m}$ to be a transformation matrix. Note: to define $\mathbf{m},$ edit the Math Box following the assignment characters, := .
- When you open the .tns file, $\mathbf{m}=\mathbf{a}$ initially.

In the right work area, grab and drag the vector $\mathbf{v}$ (at the tip of the arrow).

- The product, $\mathbf{w}=\mathbf{m}\cdot\mathbf{v},$ in the left work area, and the vector $\mathbf{w},$ in the right work area, are automatically updated.

On Page 1.3, there are several defined transformation matrices and constants.
There are also several Math Boxes on Page 1.4 to compute $\mathbf{m}\cdot\mathbf{v}$ for various input vectors $\mathbf{v}$.

Note: The calculator function **norm** of a vector returns the length of the vector. Consider how each of the following transformations affects the magnitude and direction of the input vector.

1. Let $\mathbf{m}=\mathbf{a}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}.$

   a. Describe this transformation in words.

b. Complete the following table.

v	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}0\\2\end{bmatrix}$	$\begin{bmatrix}-4\\0\end{bmatrix}$	$\begin{bmatrix}0\\-5\end{bmatrix}$	$\begin{bmatrix}2\\2\end{bmatrix}$	$\begin{bmatrix}3\\-3\end{bmatrix}$
$\mathbf{m}\cdot\mathbf{v}$						

c. Do the calculations in the table above support your description from part a? Why or why not?

2. For $\theta=\frac{\pi}{4}$, let $\mathbf{m}=\mathbf{b}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$.

Note: To change the transformation matrix, click the Math Box in which $\mathbf{m}$ is defined (on Page 1.2). Delete the current transformation matrix (for example, $\mathbf{a}$), and type the variable representing any one of the transformation matrices defined on Page 1.3 just after the assignment characters := (for example, $\mathbf{b}$).

a. Describe this transformation in words.

b. Complete the following table.

v	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}2\\2\end{bmatrix}$	$\begin{bmatrix}-4\\-4\end{bmatrix}$	$\begin{bmatrix}0\\-1\end{bmatrix}$	$\begin{bmatrix}5\\5\end{bmatrix}$	$\begin{bmatrix}\sqrt{2}\\-\sqrt{2}\end{bmatrix}$
$\mathbf{m}\cdot\mathbf{v}$						

c. Do the calculations in the table above support your description from part a? Why or why not?

d. Describe this transformation for any value of θ.

3. Let $\mathbf{m} = \mathbf{c} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

   a. Describe this transformation in words.

   b. Complete the following table.

$\mathbf{v}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} -4 \\ -6 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 7 \end{bmatrix}$	$\begin{bmatrix} -4 \\ 6 \end{bmatrix}$
$\mathbf{m} \cdot \mathbf{v}$						

   c. Do the calculations in the table above support your description from part a? Why or why not?

4. Let $\mathbf{m} = \mathbf{d} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$..

   a. Describe this transformation in words.

   b. Complete the following table.

$\mathbf{v}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} -6 \\ -4 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$	$\begin{bmatrix} -3 \\ -5 \end{bmatrix}$
$\mathbf{m} \cdot \mathbf{v}$						

   c. Do the calculations in the table above support your description from part a? Why or why not?

5. For $k = 2,$ let $\mathbf{m} = \mathbf{e} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}.$

   a. Describe this transformation in words.

   b. Complete the following table.

Note: $|\mathbf{v}|$ is the magnitude, or length, of the vector $\mathbf{v}.$ The magnitude of the vector $\mathbf{v}$ can be found on Page 1.4: $\text{norm}(\mathbf{v}) = |\mathbf{v}|.$

$\mathbf{v}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} -3 \\ 4 \end{bmatrix}$	$\begin{bmatrix} -5 \\ -12 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$
$\lvert\mathbf{v}\rvert$						
$\mathbf{m} \cdot \mathbf{v}$						
$\lvert\mathbf{m} \cdot \mathbf{v}\rvert$						

   c. Do the calculations in the table above support your description from part a? Why or why not?

   d. Describe this transformation for any value of $k > 0.$

6. Let $\mathbf{m} = \mathbf{h} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

   a. Describe this transformation in words.

   b. Complete the following table.

v	$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 7 \end{bmatrix}$	$\begin{bmatrix} -3 \\ -4 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	$\begin{bmatrix} 2 \\ -5 \end{bmatrix}$	$\begin{bmatrix} \sqrt{11} \\ 12 \end{bmatrix}$
$\mathbf{m} \cdot \mathbf{v}$						

   c. Do the calculations in part (b) support your answer in part (a)? Why or why not?