# Exploring Exponents

**TEACHER NOTES**

**MATH NSPIRED**

## Math Objectives

- Students will investigate and describe how negative exponents affect positive integer bases.
- Students will describe the effect of a zero exponent on positive integer bases.
- Students will apply abstract and quantitative reasoning to determine the numerical value of an integer exponent.
- Students will understand the structure of integer exponents by demonstrating how they can be rewritten as a series of repeated multiplication or division.

## Vocabulary

- base
- exponent
- integer

## About the Lesson

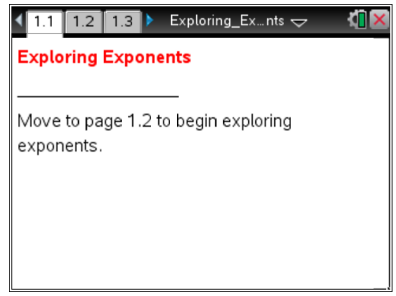
- This lesson involves preparing students to understand properties of integer exponents.
- As a result, students will:
  - Describe how negative exponents affect a positive integer number base.
  - Make and test conjectures about the effect of having a zero or negative integer exponent on a positive integer base, based upon the examples observed.
  - Use patterns from their numerical observations to support their rationale of the effect of zero and negative exponents on positive integer bases.

## TI-Nspire™ Navigator™ System

- Send out the *Exploring_Exponents.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.
- Use Quick Poll to assess students' understanding.

## Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software




**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at [http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials](http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials)

**Lesson Files:**

*Student Activity*

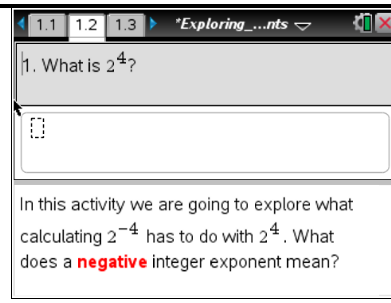
- Exploring_Exponents_Student.pdf
- Exploring_Exponents_Student.doc

*TI-Nspire document*

- Exploring_Exponents.tns

## Discussion Points and Possible Answers

**Move to page 1.2.**



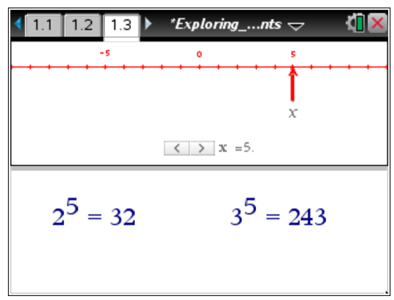

1. What is the numerical value of $2^4$? Describe how you determined this value.

   **Answer:** 16 = 2×2×2×2. It is the product of 2 with itself, four times.

> **Tech Tip:** Use [tab] to move to the answer box. The student can select [menu] to check their answer. This question could also be used as a Quick Poll.

**TI-Nspire Navigator Opportunity: *Quick Poll***
**See Note 1 at the end of this lesson.**

**Move to page 1.3.**




2. Use the arrows to increase or decrease the exponents on $2^x$ and $3^x$. Describe what you see happening to the values in the equations.

   **Sample Answer:** When you increase the exponent, the values get bigger, and when you decrease the exponent, the values get smaller. When the exponent is negative, the values change to fractions that are positive and less than 1.

> **Teacher Tip:** Initially, allow students a little time to use the arrows on the slider on page 1.3 to increase or decrease the value of the exponent. Let students talk to each other to discuss what they think the negative exponent means. After students have shared with each other, ask what they noticed when they selected on the slider.

> **Tech Tip:** Use [tab] to select the arrows. Then use the arrows on the handheld to easily change the value of $x$.

> **Tech Tip:** Students can tap in the bottom right corner to move $x$ to the right, and in the lower left to decrease the value of $x$.

3. As you increase (or decrease) the exponents, look for a pattern between the exponents and the values in the equations to help you explain why the values in the equations change as they do.

Have students complete the table on their worksheets to help them identify patterns.

| $x$ | $2^x$ | $3^x$ |
|---|---|---|
| 3 | 8 | 27 |
| 2 | 4 | 9 |
| 1 | 2 | 3 |
| 0 | 1 | 1 |
| -1 | 1/2 | 1/3 |
| -2 | 1/4 | 1/9 |

**Sample Answer:** Exponents mean repeated multiplication; you multiply the base by itself the number of times as shown in the exponent.

> **Teacher Tip:** Ask students for an explanation as to *how* the values are getting bigger and smaller. Many might notice that increasing occurs by multiplying by 2 (or 3). For example, for $3^5$, you might want students to have responses such as "You'll need to multiply 3 by itself 5 times, 3 × 3 is 9, 9 × 3 is 27, 27 × 3 is 81, and 81 × 3 is 243", to ensure they understand the meaning of exponents. Encourage as much mental arithmetic as possible. It might not be common, however, for students to notice that when the values are decreasing, they are being *divided* by 2 (or 3). You might not need to push for this insight for this particular question yet; wait and see what thinking emerges with the next question as students try to make sense of an exponent of zero.

4. Use your reasoning from question #3 to help explain why $2^0 = 1$ and $3^0 = 1$.

**Sample Answer:** $2^5 = 32$, $2^4 = 16$; the value of the exponent decreased by one and 16 is 32 divided by 2. $2^3 = 8$; the value of the exponent decreased by one and 8 is 16 divided by 2. Continuing this pattern, $2^2 = 4$ and $2^1 = 2$. Decreasing the exponent by one more and dividing 2 by 2, we find $2^0$ has to be 1.

> **Teacher Tip:** Encouraging the students to make this argument is ideal. A mathematical principle is defined using a consistent pattern.

You can ask if the two expressions, $2^x$ and $3^y$, always provide different values and why. Since one expression only has factors of 2 and the other, only factors of 3, they will never have a common value, except for when the exponent is zero.

Mathematically proficient students should be able to describe the numerical pattern connecting the decreasing exponents and the corresponding values of the exponential expressions. This pattern provides a structure from which to reason that any positive integer base value raised to a power of zero has to have a value of 1. This reasoning will preserve the structure they have found in understanding exponents and can also be referred to when working with negative exponents in the following questions.

5. What do you notice about the values in the equations when the exponents are negative?

**Sample Answers:** When the exponents of a positive integer are negative, the values in the equation are fractions that are positive and less than 1.

**Teacher Tip:** The intent is that students notice the connection between the values with corresponding positive and negative exponents, e.g., $3^3 = 27$ and $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

**TI-Nspire Navigator Opportunity: *Live Presenter***
**See Note 2 at the end of this lesson.**

6. James says that $4^{-2} = -16$. Kayley says that $4^{-2} = -1/16$. With whom do you agree and why?

**Sample Answers:** Neither is correct. $4^{-2} = 1/16$. James and Kayley do not realize that the negative in the exponent does not make the numerical expression negative. Another way to understand this is $4^5/4^2 = 4^3$, so similarly $4^2/4^5 = 4^{-3}$ or $\frac{1}{4^3}$.

**Teacher Tip:** This question is important because it challenges students to construct a viable argument. Take time to discuss the various answers.

7. What is the value of $2^{-11}$? Would $2^{-12}$ be greater or less than $2^{-11}$? Explain your reasoning. .

<u>**Answer:**</u> $2^{-11}$ is $\frac{1}{2^{11}} = \frac{1}{2064}$. $2^{-12}$ is $\frac{1}{2^{12}} = \frac{1}{4096}$, which would be smaller than $2^{-11}$ because there is a larger number in the denominator of the unit fraction.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The effect of negative exponents on positive integer bases.
- The effect of a zero exponent on positive integer bases.

> **Teacher Tip:** Students will reason abstractly and quantitatively. Students' original thinking on exponents will most likely connect to repeated multiplication. However, when thinking about negative and zero exponents, mathematically proficient students' conceptions need to broaden to a more abstract representation of numbers. For example, $\frac{2^3}{2^8} = \frac{1}{2^5} = 2^{-5}$. In this example, we cannot think of multiplying 2 by itself "-5" times. However, $2^{-5}$ is a numerical representation using exponents for the number $\frac{1}{2^5}$ or $\frac{1}{32}$.

## Assessment

**TI-Nspire Navigator Opportunity:** ***Quick Poll (Open Response)***

**See Note 3 at the end of this lesson.**

1. Ask students to predict the values of expressions such as $5^{-3}$, $8^{-2}$, and $9^{-6}$ (one at a time), and use the Quick Poll to show student responses.

<u>**Answer**</u>: $\frac{1}{5^3} \text{ or } \frac{1}{125}; \frac{1}{8^2} \text{ or } \frac{1}{64}; \frac{1}{9^6} \text{ or } \frac{1}{531441}$.

2. Let *a* be a positive integer. Show why $a^0$=1.

<u>**Sample Answers**</u>:

$a^3 = a \cdot a \bullet a$

$a^2 = a \bullet a$

$a^1 = a$

$a^0 = 1$

## TI-Nspire Navigator

**Note 1**

**Question 1, Quick Poll**

Send the question on page 1.2 as a Quick Poll for a bell ringer or to check for foundational understanding.

**Note 2**

**Question 4, Live Presenter**

This is a moment to use Live Presenter to have a student (or more than one student to allow for students to explain different thinking) describe their thinking regarding negative exponents as they refer to Page 1.3.

**Note 3**

**Assessment, Quick Poll**

A Quick Poll can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students might have.