Ride the Rollercoaster

About the Lesson

In this activity, students will use polynomial regression to develop and assess how well the regression equations model the given set of data. As a result, students will:

- Evaluate the equation models for reasonableness in their use for extrapolating beyond the given data sets.

Vocabulary

- cubic regression
- quartic regression
- coefficient of regression

Teacher Preparation and Notes

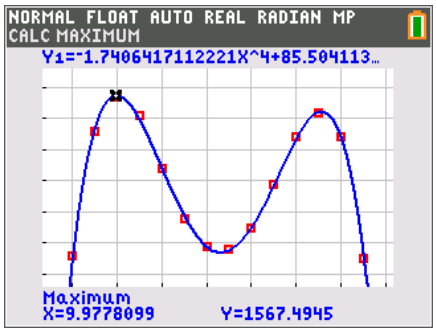
- Problem 1 of this activity may be done in class while Problems 2 and 3 could either be done in class or assigned as homework depending on time available.
- As an extension, the teacher could also include relative and absolute maxima and minima.
- The data for Problems 1 and 2 are not actual data values, but have been generalized for visual reference.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

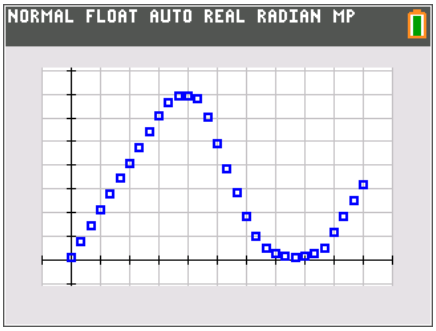
Lesson Files:

- Ride_Rollercoaster_Student.pdf
- Ride_Rollercoaster_Student.doc
- L1.8xl
- L2.8xl
- L3.8xl
- L4.8xl
- L5.8xl
- L6.8xl

Tech Tip: Before beginning the activity, the lists L_1, L_2, L_3, L_4, L_5 and L_6 need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ Ce Software.

Problem 1 – Ride the Rollercoaster

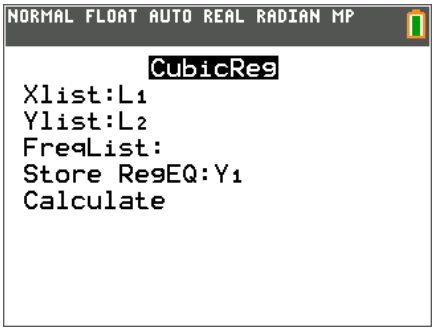
This problem involves developing a regression equation to model a section of track for a steel rollercoaster. Using the given data, students first create a scatter plot and, using their observations of the graph shape, choose a regression method and graph the resulting equation on the scatter plot.



Tech Tip: If your students are using the TI-84 Plus CEhave them turn on the GridLine by pressing 2nd zoom[format] to change the graph settings. If your students are using TI-84 Plus, they could use GridDot.

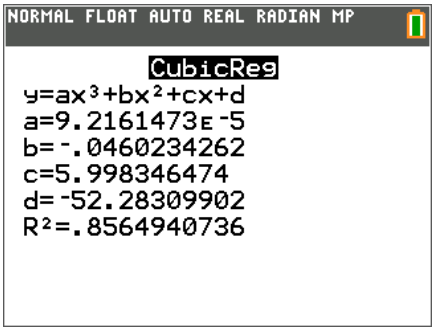
1. Based on the scatter plot's appearance, what type of polynomial equation might reasonably be chosen to represent these data?

Answer: A cubic equation



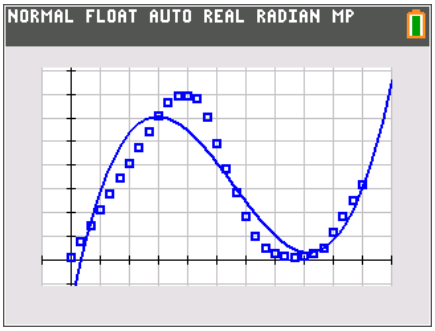
Based on the "N" shape of the graph, students will likely be drawn to perform a cubic regression. If students do not have experience with general polynomial graph shapes, it will be helpful to direct the students to use cubic regression for the first graph.

The sequence for entering the regression can be a bit complicated for students. Make sure students observe the teacher's viewing windows as the keys are pressed to help with this.



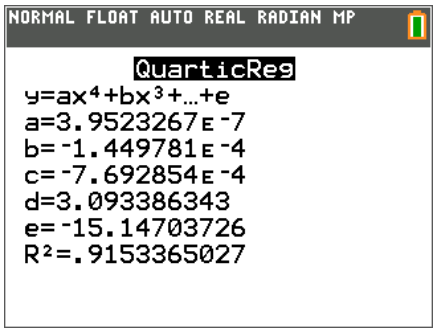
2. What equation resulted from your first regression choice? How did it fit the data?

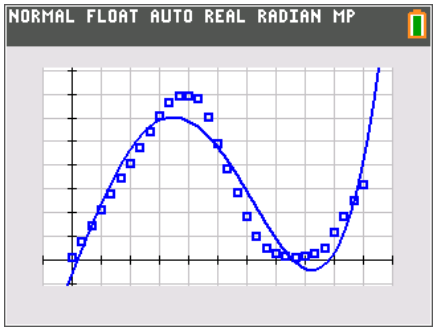
Answer: A cubic regression didn't seem to fit the data points too well.



Students are next asked to try another regression model and to graph it also on the scatter plot.

For the second model attempt, students may explore models, or the instructor may prefer to direct the students to use a quartic model. This model is graphed to the right.





3. What equation resulted from your second regression choice? How did it fit the data?

Sample Answer: Typically any regression option other than quartic will result in a poor correlation.

4. After viewing the two equations graphed on top of the scatter plot, which appears to best fit the data?

Answer: The quartic regression equation.

Students are to make a visual comparison of the results to assess visually which equation model appears to best fit the data. Then they are directed back to the pages with the calculated regressions to compare the values of R^2 to assess the fit.

5. Compare the values of R^2 for the two regressions. Based on these values, which equation provides the best fit to the rollercoaster data?

Answer: The quartic regression model gives an R^2 value of 0.9153 where the cubic regression model gives an R^2 value of 0.8565.

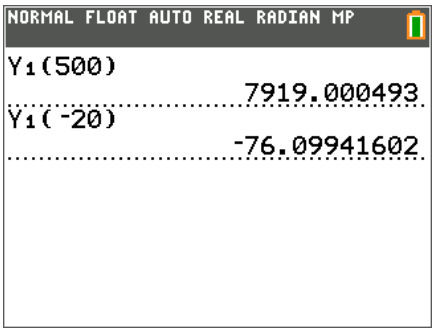
6. Calculate the maximum height you can expect to reach on this portion of the rollercoaster track by pressing 2nd trace [calc] and selecting **maximum**.

Answer: ≈ 181.4 feet

7. When you've travelled 500 feet horizontally, what height do you expect according to your equation? Does this make sense? Explain.

Sample Answer: According to the quartic model, $f(x) \approx \left(3.95 \times 10^{-7}\right)x^4 - \left(1.45 \times 10^{-4}\right)x^3 - \left(7.69 \times 10^{-4}\right)x^2 + 3.09x - 15.15$; $f(500) \approx 7919$ feet; This doesn't make sense because that would be over one mile high!

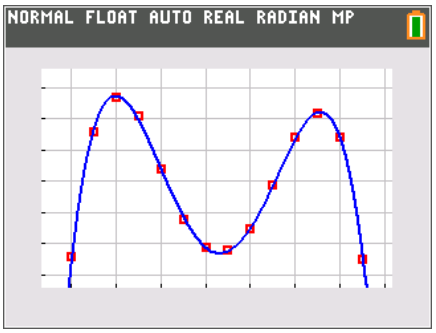
8. If the rollercoaster were to roll backward, what would you expect the height to be 20 feet back ($x = -20$)? Does this make sense? Explain.



Sample Answer: ≈ –76.1 feet; This does make sense unless the rollercoaster is allowed to go underground.

Problem 2 – The Financial "Rollercoaster"

Students explore NASDAQ index data for a one-day period and choose an appropriate polynomial regression model to fit the data.



It may be helpful to discuss indices such as NASDAQ, S&P 500, and the Dow Jones to assess economic health.

Lists **L3** and **L4** contain data (time in hours after 9:00 a.m. and number of shares traded, respectively) which model the NASDAQ stock market performance indicator data for one day in November of 2008.

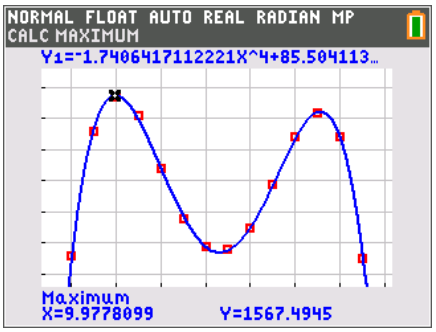
9. Based on the scatter plot appearance, what type of polynomial equation might reasonably be chosen to represent these data?

Answer: A quartic polynomial.

10. What regression equation best fits the given NASDAQ data? Why?

Sample Answer: $f(x) = -1.7406x^4 + 85.5041x^3 - 1556.8463x^2 + 12446.6333x - 35311.9226$; This quartic regression model seems to pass through a majority of the data points and only narrowly misses the points through which it doesn't pass. This gives it an R^{2-}value close to 1.

11. A high NASDAQ index value generally indicates economic health. At what time during this day was the index most favorable? What was the approximate index value at this time?



Answer: At just before 10:00 a.m. (approx. 9:58), approximately 1567 shares were traded.

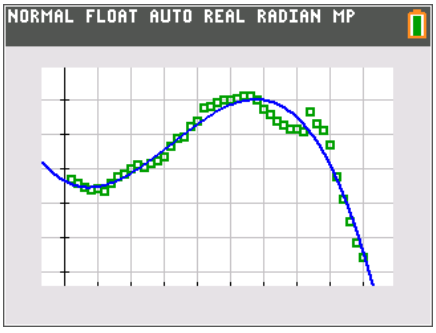
12. If this data set gave index values for today, based on the graph and the equation you found to represent the data, what kind of values should be expected for tomorrow? How about yesterday? Is this reasonable? Explain.

Sample Answer: According to the graphed mode, the NASDAQ index is rapidly decreasing as of the end of the day in November of 2008. By the time the graph was extrapolated to yesterday's date or today's date, the NASDAQ index would be a very large negative value which is impossible.

Problem 3 – The Gas Prices "Rollercoaster"

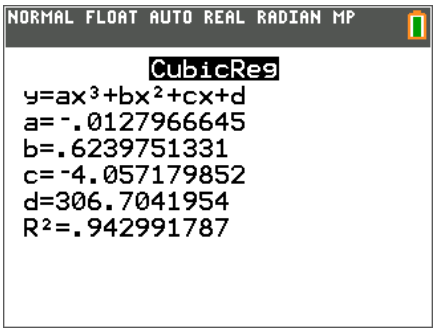
Students explore the changes in regular gasoline prices given average weekly prices for gasoline beginning with the first week of January 2008, and choose an appropriate polynomial regression model to fit the data.

This data could interest students in looking at other interesting current economic trends. Students could explore trends such as SUV sales, fuel- efficient car purchases, food prices, and unemployment data for the current year.



13. Based on the scatter plot appearance, what type of polynomial equation might reasonably be chosen to represent these data?

Answer: A cubic equation

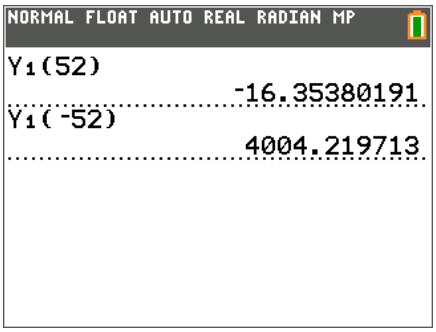


14. What regression equation best fits the given gasoline price data? Why?

Answer: $f(x) = -0.0128x^3 + 0.624x^2 - 4.0572x + 306.7042$; Its R^2–value is the closest to 1.

15. Based on your equation model, what price might be expected for gasoline during the last week of the year (week 52)?

Answer: ≈ –16 cents



16. Based on this model, what would the price of gas been at the beginning of 2007 ($x = -52$)?

Answer: ≈ $40.04. Ouch!

17. While the regression equation obtained may provide a reasonable fit to the given data, what problem(s) would be involved with extrapolating beyond the given data values?

Sample Answer: The graph and the equation give the impression that as weeks go on, the price will continue to drop to the point where it will be free and when the price is negative, it would imply that the customer would be given money to take the gas. While this would be nice, it is not realistic. Similarly, if we look back to weeks prior to January of 2007, the impression is given that the price of gas would become infinitely high, which is not historically accurate. In the 1980's, gas was around a dollar a gallon.

Stress with students that the models used in this activity work for the given data and extrapolation is often problematic, so the application of such equations modeling real situations typically involves a restricted domain.