Welcome to the MATRIX

Transforming 2D Space

Teachers Teaching with Technology™
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Student Activity

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TI-Nspire™ Investigation Student 45 min

Introduction

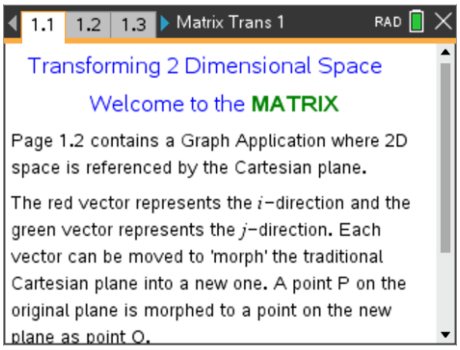
The QR codes included here link to three videos related to this activity. The presenter, Grant Sanderson, has produced an entire series on this topic. You don't need to do any calculations whilst watching these three videos, they will however provide an insight to the content in this activity. Scan the QR codes and watch the videos.

Essence of Linear Algebra | Combinations, span and basis vectors | Linear Transformations (matrices)

Transforming 2 Dimensional Space

Open the TI-Nspire file: Matrix Trans 1

Page 1.1 contains a brief introduction. Read the instructions then navigate to page 1.2.



The red vector is a transformation of the unit vector: $\hat{\underset{\sim}{i}}$.

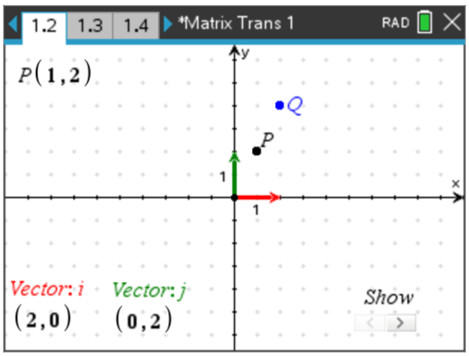
The green vector is a transformation of the unit vector: $\hat{\underset{\sim}{j}}$

Point P exists on the Cartesian plane. The coordinates of point P are displayed in the top left corner of the screen.

Point Q is the image of P using the transformation created by the combination of the red and green vectors.

The coordinates for the red and green vectors are displayed in the bottom left corner of the screen. These two vectors can be used to transform the plane.

In the screen shown opposite, the plane has been dilated by a factor of 2 in both the x and y direction.



Question: 1.

Suppose point P is moved to (2, 3). Using the same transformation as above, what would be the location of Q?

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Question: 2.

Suppose point P is at (x, y) and undergoes the same transformation as above, such that point Q is (-6, 8). What would be the coordinates of point P?

Question: 3.

Change vector $\underset{\sim}{i}$ to a length of 4 units, in the positive x direction (only) and vector $\underset{\sim}{j}$ to a length of 3 units, in the positive y direction (only). With point P at (1, 2), what are the coordinates of point Q?

Question: 4.

Change vector $\underset{\sim}{i}$ to a length of 2 units, in the positive x direction (only) and return vector $\underset{\sim}{j}$ to a unit length (1), in the positive y direction (only). Point P can be placed in each of the locations below.

a) For each location, determine the corresponding coordinates of the transformed point: Q.

P: (x, y)	(-2, 4)	(-1, 1)	(0, 0)	(1, 1)	(2, 4)
Q: (x, y)					

b) The location of each point P (above) could be described by a function: $y = x^2$.
Study the points generated for Q and determine an appropriate function to describe the location of these transformed points.

At the conclusion of this question, return vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ to 2 units each in the x and y directions respectively.

The world according to Q.

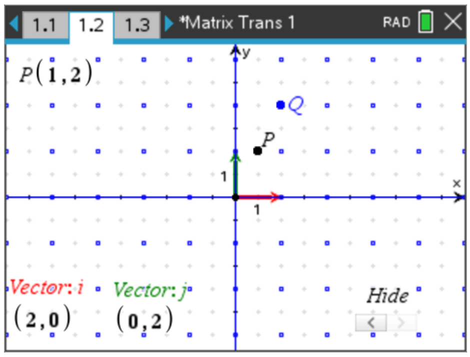
The show/hide toggle in the bottom right section of the screen allows you to view the transformed plane, "Q's World".

The blue points show how the original 1 x 1 grid has been stretched in both the x and y direction, each by a factor of 2, just like a scaled drawing.

The coordinates of point Q on the *transformed plane* are: (1, 2), it's position relative to the transformed plane is the same as P on the original plane. Point Q is the image of point P under the transformation of the plane.

In mathematics we are generally interested in the coordinates of Q, relative to the original plane. In reference to the original plane, Q is located at the point: (2, 4).

It is useful to remember that in Q's world, (the transformed plane), it's relative position is unchanged.



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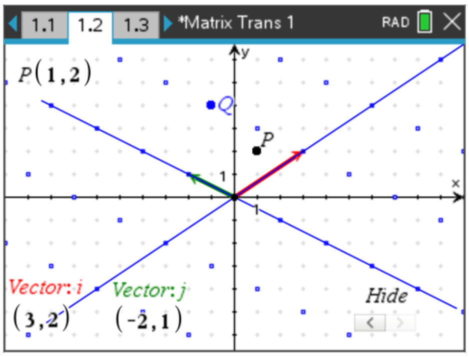
Vectors *i* and *j* can *skew* the plane. In the diagram shown here, vector *i* has been moved to (3, 2) and vector *j* to (-2, 1). Relative to the skewed plane, Q is still located at: (1, 2).

Relative to the original cartesian plane, Q is located at: (-1, 4)

The transformation can be summarised by a matrix that contains the two vectors:



vector $\underset{\sim}{i}$	vector $\underset{\sim}{j}$	Transformation Vector
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$

When the transformed plane is displayed (toggle on), content connected to the *i* and *j* vectors incorporates many calculations slowing the calculator down … ☹ ☺ ☹.

The calculator will respond quicker if the transformed plane is hidden while vectors are being moved.

Question: 5.

The point P can be expressed as a column vector $\begin{bmatrix} x \\ y \end{bmatrix}$.

a) Determine the product of the transformation matrix: $\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$ and point P $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$; comment on the result.

b) Supposed point P is moved to (2, 3) and undergoes the same transformation, determine the new location of point Q (the image of P) under this transformation using matrices and the interactive TI-Nspire diagram.

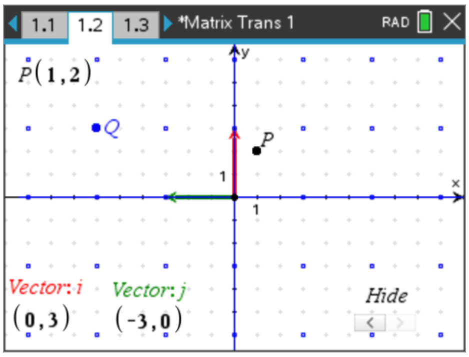
c) Point P is located at $\begin{bmatrix} x \\ y \end{bmatrix}$ and undergoes the same transformation as above such that Q is located at (9, -1).

Determine the location of point P.

In the image shown here, the $\hat{\underset{\sim}{i}}$ vector has been moved onto the y axis and dilated by a factor of 3. At the same time the $\hat{\underset{\sim}{j}}$ vector has been moved to the negative x axis and also dilated by a factor of 3.

This transformation can be described numerically using the matrix:

$$\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$$



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Question: 6.

a) Determine the location of point Q using the transformation matrix (given above).

b) Compare the distances: $|OP|$ and $|OQ|$.

c) Describe the transformation geometrically.

Question: 7.

The P (x, y) is transformed such that $Q\left(\begin{bmatrix} x' \\ y' \end{bmatrix}\right) = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ and Q is located at: (-7, -1):

a) Determine the location of P.

b) Suppose point P moves along the line: $y = x + 1$, describe the corresponding location of Q.

c) Suppose point P moves along the x axis, describe the corresponding location of Q.

Question: 8.

The P (x, y) is transformed such that $Q\left(\begin{bmatrix} x' \\ y' \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$:

a) Move point P and describe the corresponding locations for Q.

b) Explain what has happened geometrically.

c) Point Q is located at (2, 4). Determine the location(s) for point P.

d) Identify another transformation that would have a similar impact.

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