Amazing Chords

Teachers Teaching with Technology™
Professional Development from Texas Instruments

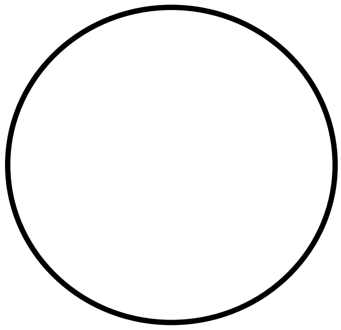
Student Activity

7 8 9 10 **11** 12

TI-Nspire Investigation Student 60 min

Introduction

Imagine a circle of radius one unit. Draw a chord (line connecting two points on the circumference of the circle) anywhere on the circle, then measure the length of the chord. Now imagine your friend also draws a chord somewhere else on the circle. Suppose you continue to draw chords on the circle, what would be the average length of these chords?

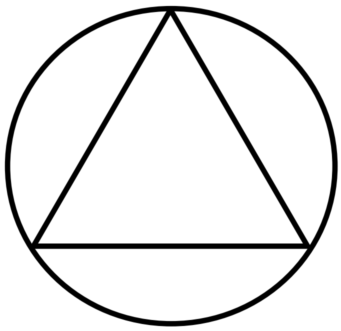


Question: 1.

Measure the diameter of the circle (opposite) and draw ten chords on the circle. Measure the length of each chord.

a) What is the average chord length?

Calculating the theoretical average chord length is somewhat challenging. The question can be made easier by adding some detail. An equilateral triangle has been drawn in the circle opposite. The aim is to compare the length of a randomly drawn chord to the side length of the equilateral triangle.



Question: 2.

What proportion of your randomly drawn chords are longer than the side length of the equilateral triangle? (Both circles are the same size)

A small sample of size 10 is relatively small. To speed things up a couple of programs have been written into a TI-nspire™ to generate lots of chords. The following instructions provide a guide to work through the simulated and theoretical results to compute the proportion of random chords that are longer than the side lengths of the inscribed equilateral triangle.

Chord - 1

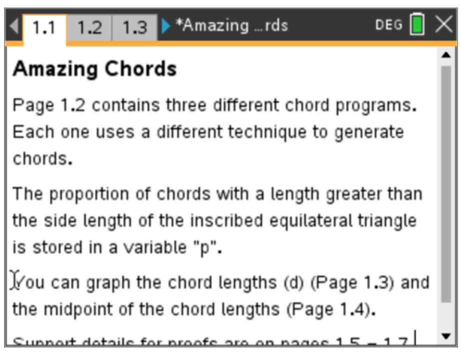
Instructions:

Open the TI-Nspire file: Amazing Chords.

Navigate to Page 1.2 and run the "Chord1" program.

To run the program press the [var] key and select Chord1. Press [enter] to run the program.

A prompt will appear requesting the number of chords to be generated. Start with 100 chords.



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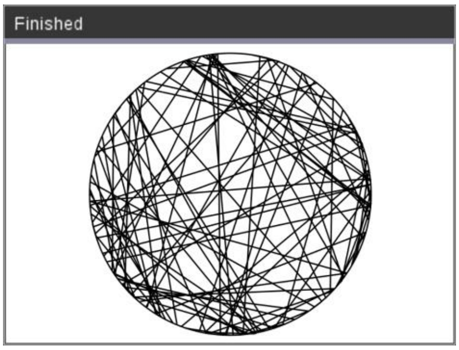
The screen shown opposite shows 100 randomly generated chords (program output). Press esc to return to the calculator application.

To see the proportion of chords that are longer than the side length of the inscribed equilateral triangle, press “P” followed by enter.

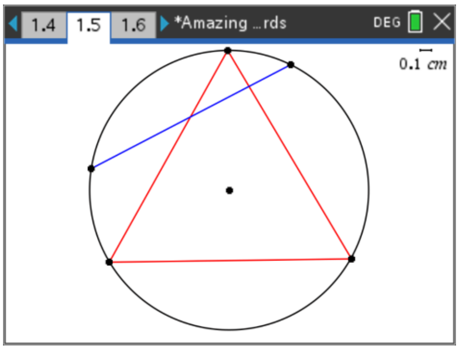
The length of each chord is stored in a list “d”. The diameter of the circle in the simulation is 200 units. The distribution of the chord lengths can be seen on page 1.3.



Question: 3.

Run the program 10 times and record the proportion of chords that are longer than the side length of the equilateral triangle (p). Based on your results, what do you think the probability that a randomly generated chord will be longer than the side length of the inscribed equilateral triangle?

Navigate to page 1.5. A circle with radius 1 unit has been drawn. An equilateral triangle has been inscribed in the circle, so too a moveable chord.



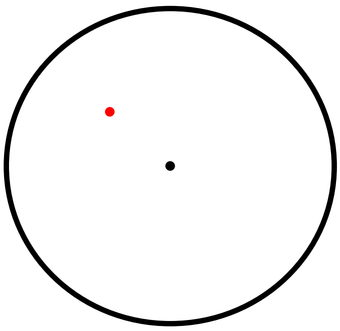
- Move the chord to wherever you want on the circle.
- Drag one of the triangle vertices so that it is on top of one end of the chord.
- Study this diagram carefully. Move the end of the chord that is not aligned to the triangle.
- Without measuring, how can you determine if your ‘random’ chord is longer or shorter than the side length of the triangle?

Question: 4.

Based on your experimentation (above), what proportion of randomly generated chord lengths will be longer than the side length of the inscribed equilateral triangle? [Justify your answer]

Chord – 2

Another way to produce random chords is to place a point at some random location inside the circle and draw a chord through this point. For the purpose of this section, assume the random point represents the middle of the chord.

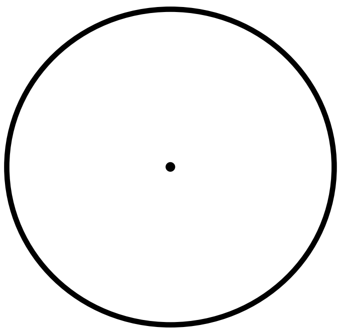


Question: 5.

Draw a diagram to help explain how a chord can be drawn such that the randomly generated point is at the midpoint of the chord.

Question: 6.

Plot 5 points inside the circle and draw chords through each such that each random point is located at the midpoint of the corresponding chord.



a) Record the lengths of the chords.

b) What proportion of your chords are longer than the side length of an inscribed equilateral triangle? (Circle is the same size as Question 1 & 2)

c) Based on your sample, are both methods of generating chord lengths producing similar results?

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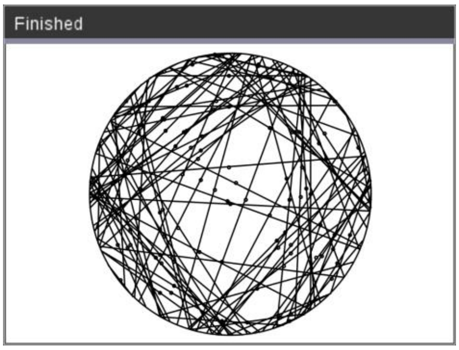
Return to Page 1.2 and run the chord2() program. Generate 100 random chords. Press esc to return to the calculator application.



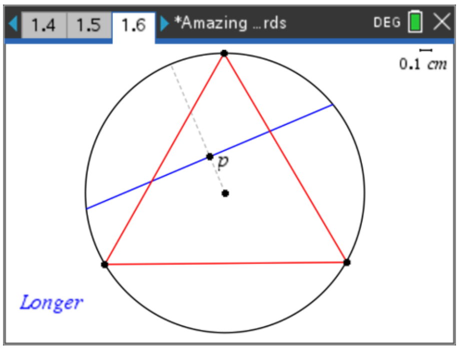
To see the proportion of chords that are longer than the side length of the inscribed equilateral triangle, press "P" followed by enter.

The length of each chord is stored in a list "d". The diameter of the circle in the simulation is 200 units. The distribution of the chord lengths can be seen on page 1.3.

Question: 7.

Run the program 10 times generating 100 chords each time. Record the proportion of chords that are longer than the side length of the equilateral triangle (p). Based on your results, what do you think the probability that a randomly generated chord will be longer than the side length of the inscribed equilateral triangle using this approach?

Navigate to page 1.6. Point P is the random point. The red triangle is an equilateral triangle inscribed in the circle. The length of the chord changes automatically as P is moved around.



The text in bottom left corner of the screen indicates when the chord is *longer* than the side length and also *shorter*.

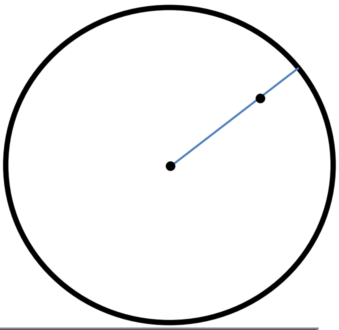
Move point P around the interior of the circle and establish an 'artificial' boundary whereby all chords generated through P will be longer than the side lengths of the triangle.

Question: 8.

Through what region can point P move to ensure all chord lengths generated in this region will be longer than the side lengths of the triangle?

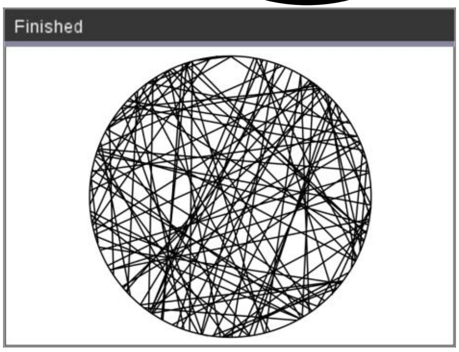
Chord – 3

Another way to produce random chords is to draw a line from the centre of the circle to the circumference. Place a point on the line and have it move along the line to any random location, once again the point represents the midpoint of the chord. The radial arm can be rotated everywhere so that any chord can be generated. This option appears to be remarkable similar to the Chord-2 example.



Navigate to page 1.2.

For this example, progress straight to the simulation. Run the chord-3 program and determine the proportion of chords that are longer than the side length of the equilateral triangle.



Chords in this example are placed randomly along the radial arm from the centre of the circle to the circumference.

A sample of 100 randomly generated chords using this method is shown opposite.

Question: 9.

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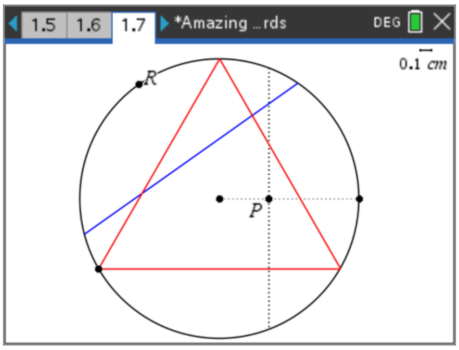
Use the simulation to determine the approximate proportion of chords that are longer than the side length of the inscribed triangle.

Navigate to page 1.7. Point P is a random point along the radius and represents the midpoint of the chord. This chord can then be rotated using point R.

Move point P along the radius and observe 'ghost' chord (dotted) and the final chord after rotation (blue).



Question: 10.

For what range of points along the radius will point P produce a chord that is longer than the side length of the inscribed triangle?

Question: 11.

Assuming that point P is equally likely to land on any point along the radius, what proportion of chords produced this way will be longer than the side length of the inscribed triangle?

Crazy Chord Discussion

In this activity you have explored three different ways to generate a random chord in a circle and compare the length to the side length of an inscribed triangle. The geometrical 'proofs' produce results consistent with the simulations. Run the different chord#() programs again and explore the distribution of the corresponding lengths created (Page 1.3) and corresponding midpoint distribution (Page 1.4).

Discuss: When a chord is randomly generated in a circle, the probability that it is longer than the side length of an inscribed triangle is …

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