Fair Go Penny

Investigation

Alex challenges his friend Gustav to a simple two player game that involves tossing a coin. Player A nominates a head / tail sequence, consisting of three outcomes. Player B can then nominate their head / tail sequence consisting of three consecutive outcomes. The coin is tossed repeatedly until one of the nominated outcomes is achieved.

Gustav doesn't seem interested in playing, he claims: "It's a silly game… if I choose Head, Tail, Head (HTH), it has a 1/8 chance of coming up, it doesn't matter what sequence you choose, it too will have a 1/8 chance of coming up." Alex is keen to play the game so he offers a wager as an incentive: "What if we play for money, best of 10 games, $2.00 per win" says Alex. Gustave agrees to play so the two boys make their selections:

> Gustav chooses the sequence: Head, Tail, Head. (HTH)

> Alex then chooses the sequence: Head, Head, Tail. (TTH)

Question: 1.

> If the boys play 10 games who do you think will end up with the most money?

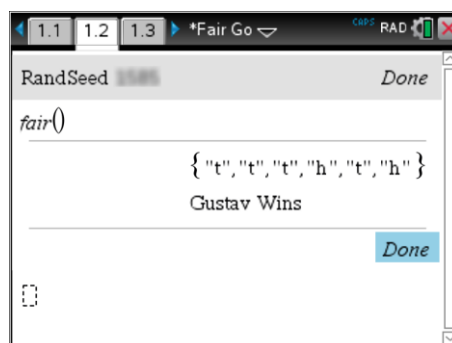The game can be simulated on the calculator.

Instructions:

Open the TI-nspire file: "Fair Go" and Navigate to page 1.2.
A **single** game can be played. The results will be displayed as follows:

Event – List of coin toss results until a win was obtained.

Outcome – Name of winner

Before commencing **any** probability simulation, it is important to 'seed' the random variable generator. The random seed command can be obtained from the calculator application:

> \> **Probability** > **Random** > **Seed**

Enter a unique four digit number such as the last four digits of your home or mobile number to ensure your results are also unique. To play a single game, press the [**VAR**] key, select the game and press [**ENTER**].

Question: 2.

> Play the game 10 times and record the results in a table, A when Alex wins and G for Gustav. Alex's Balance is based on receiving $2.00 from Gustav if Alex wins or a $2.00 payout each time Gustav wins. Determine which player (if any) is the winner and comment on the results.

Game	1	2	3	4	5	6	7	8	9	10
Winner										
Alex's Balance										

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TEXAS INSTRUMENTS

Playing Detective

When the boys played, Gustav only won 3 games and subsequently claimed: "This game is not fair!" Playing 10 games does not constitute significant evidence to support Gustav's claim, his experience may be attributed to *chance*. Mathematicians make claims based on clearly defined boundaries and a level of certainty rather than reactionary emotions. One way to increase the level of confidence to accept or refute Gustav's claim is to conduct more trials (games).

Gustav decides to hire you as his 'maths-detective'. Your task is to determine whether Alex's game is fair. If the game is fair, the long term expected payout for both players is approximately $0.00, that is to say, neither player should *expect* to walk away with more money than their opponent.

As the mathematics detective, you decide the easiest way to investigate the fairness of the game is to simply play more games. In the real world, data collection generally costs money. The more data you collect the more it is likely to cost. Keep this in mind for the next part of the investigation. When prompted to simulate 'lots of games', consider what is reasonable enough to provide the evidence rather than simulating millions of games, which by the way will take the calculator a VERY long time!

Navigate to page 2.1 and select the simulation: fairmore() from the [**VAR**] menu.

This simulation requires you to select the number of games to be played, individual game results will not be displayed, rather the total number of games won by each player and Alex's balance based on the $2.00 payout each time Gustav wins and a $2.00 collection when Alex wins.
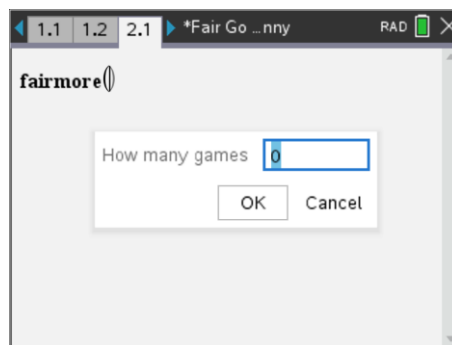
Question: 3.

Run the fairmore() simulation and select an appropriate number of games that you believe would need to be played in order to determine whether the current scenario is fair.

i)   How many games did you choose to simulate, justify your selection.

ii)   What was the outcome? Do you believe the game to be fair under its current payout system?

Question: 4.

Suppose 50 games represent one sample. If you repeated these samples over and over again. What would the distribution of the collected sample means look like? Explain.

Navigate to page 3.1 and read the instructions. After reading the instructions, navigate to page 3.2.

Click on the slider a couple of times. Each time the slider is clicked another 50 (default value of SS) games are played and the average payout recorded.

Select the slider and press: [ctrl] + [menu]

From the contextual menu, select '**Animate**'.

When one of the columns reaches approximately 50 repeat the above process and select **Stop** animation.

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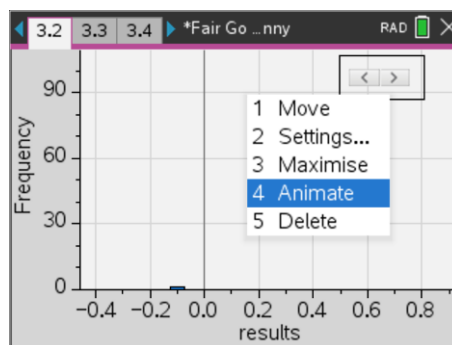
Question: 5.

How does the distribution of sample means (expected values) compare with your prediction in Question 4?

Question: 6.

As Gustav's detective, it's now time to provide him with your report. Write a brief report for Gustav identifying whether the game is fair.

Extension: Checking for Errors

In a statistical context an error is different than a mistake! In statistics there are two basic types of errors:

Type I   Reject the Null Hypothesis even though it is true.

Type II   Accept the Null Hypothesis even though it is not true.

As Gustav's detective we might start with the hypothesis that the game is fair, we would call this the Null Hypothesis. Gustav jumped to the conclusion that the game was unfair based on his experience, his data. Gustav rejects that the game is fair, he rejects the Null Hypothesis. If the game is fair then we could describe this as a Type I error. On the other hand, if our data did not provide sufficient evidence of any bias in the game, we would conclude the game is fair, but maybe the game is not actually fair. The following information helps explain how these errors occur.

The two graphs show the distribution of wins for a fair game (blue graph) and unfair game (red graph). The shaded regions under the blue graph (A & B) represent unlikely results, such as Alex wins 9 games out of 10 (B) or Gustav wins 9 games out of 10 (A). We know that these events can happen by chance in a fair game. What if we don't know if the game really is fair?
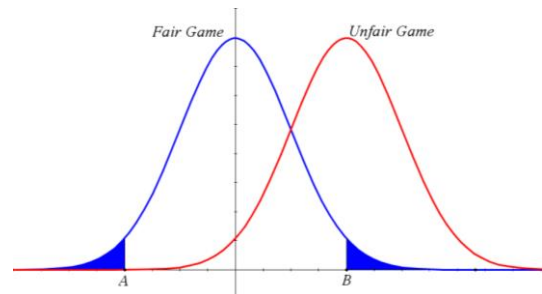
The second pair of graphs includes a 'result'. This may correspond to Alex winning 6 games from 10 in a fair game. On the other hand, it could represent 6 wins from 10 in an unfair game where Alex is expected to win 8 out of 10 games!
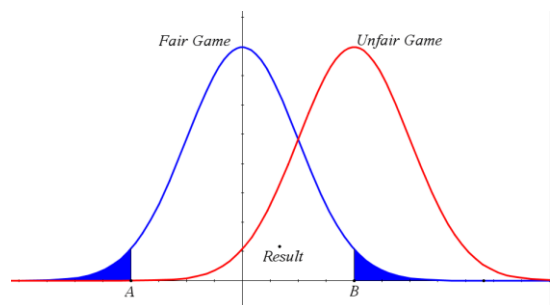
If we don't know if the game is fair, the result shown seems 'reasonable' enough to belong to the fair game. Indeed, it appears more likely to have come from the fair game than the unfair game as it is closer to the expected result (middle).

If it turns out the game is unfair, we will have experienced a Type II error. We accepted the game as fair even though it was not. Now imagine that the 'result' was in the blue shaded area (B). This is the time when we get suspicious and may accept that the result is more likely to have come from an unfair game. It is however possible that we are mistaken and jumping to conclusions. If we conclude the game is unfair, based on our result, but it did actually occur by chance, this would be a Type I error.

Question: 7.

Generating larger samples makes the graphs shown above 'narrower'. Explain how this might help reduce the chances of jumping to the wrong conclusion (Type I or Type II error).

Question: 8.

Clear the data from spreadsheet on page 3.4. Change the sample size (SS) value on page 3.1 to 100 and run the simulations again. Explain how changing the sample size (number of games in each sample) will help determine whether the game is fair or not.

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