Math Objectives

- Students will modify a piecewise linear graph of velocity to model a scenario.
- Students will make connections between a graph of an object's velocity and a corresponding graph of an object's position.
- Students will create velocity graphs based on an object's position, and position graphs based on an object's velocity.
- Students will model with mathematics (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

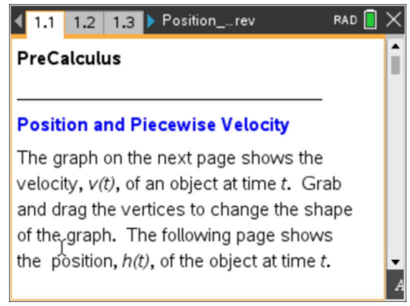
- vertex
- velocity
- position
- increasing and decreasing functions

About the Lesson

- This lesson involves creating and comparing graphical representations of velocity and position based on real-life scenarios.
- As a result, students will:
  - Manipulate segments of a piecewise linear function to represent a real life scenario about the velocity of an object.
  - Manipulate segments of a piecewise linear function to represent the velocity of an object given a real life scenario about its position.
  - Make and evaluate predictions about the position of an object given information about the object's velocity.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity
Position_and_Piecewise_Velocity_Student.pdf
Position_and_Piecewise_Velocity_Student.doc

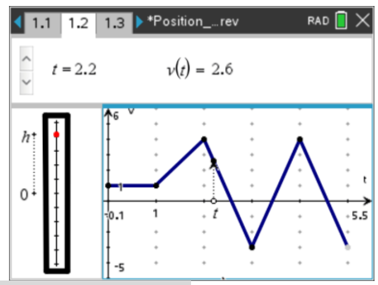
TI-Nspire document
Position_and_Piecewise_Velocity.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand () getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press ctrl to grab the point and close the hand ().

Move to page 1.2.



1. Drag the vertices on the graph so that your function goes through the following points: (0, 1), (1, 1), (2, 4), (3, -4), (4, 4) and (5, -4). This graph represents the velocity of an elevator ascending or descending floors in a building.

> **Tech Tip:** If the point which moves with the clicker coincides with one of the vertices, students will not be able to move the vertex. Move the point that coincides with the clicker away from the vertex to allow movement of the vertex.

> **Teacher Tip:** Teachers might want to have students discuss the expected motion of the elevator given this velocity. Why would an elevator be speeding up or slowing down, and why would velocities be positive or negative? How might this correspond to a real life situation? Teachers might want to remind students that an elevator would not suddenly stop in ascent, but rather slows (even if it is quickly) to a stop once it reaches a desired floor.

2. Grab the open point **t**, and drag it to the left and right.
   a. The red point on the vertical bar represents the elevator. As you move point P, what do you notice about the elevator?

   Sample Answers: The red point rises and falls as point *t* moves. When the value of the velocity function is positive, the red point rises; and when the velocity is negative, the red point falls.

b. On which time intervals is the elevator moving up? What do you notice about the velocity on these time intervals?

Sample Answers: The elevator is moving up on the interval (0, 2.5) and on (3.5, 4.5). The velocity is positive on those time intervals.

c. On which time intervals is the elevator moving down? What do you notice about the velocity on these time intervals?

Sample Answers: The elevator is moving down on (2.5, 3.5) and on (4.5, 5). The velocity is negative on those time intervals.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

d. When is the velocity 0 floors/sec? What is happening to the elevator at those times?

Sample Answers: The velocity is 0 at 2.5 seconds, 3.5 seconds, and 4.5 seconds. At those times, the elevator is changing from ascending to descending, or from descending to ascending.

3. a. When does the elevator seem to be moving the fastest? What do you notice about the velocity graph when the elevator is moving fastest?

Sample Answers: The elevator is moving fastest at 2, 3, 4, and 5 seconds. These are the values of velocity with the largest magnitude.

b. When is the elevator moving the slowest? How do you know?

Sample Answers: The elevator is moving slowest at 2.5, 3.5, and 4.5 seconds, when the velocity is 0.

Teacher Tip: If students in your class have not taken physics, you might want to point out to them that velocity is the same as speed with direction. You might also want to demonstrate that a person moving 5 mph away from a point and another person moving 5 mph toward the point have the same speed, but different velocities. Teachers might want to remind students that the dependent variable gives the object's velocity at a particular moment in time, and that velocity is a directional measure of speed. Therefore, 4 floors/second and -4 floors/second indicate that the elevator is moving at the same speed, but ascending in the first case and descending in the second.

c. When is the elevator speeding up? When is it slowing down? How do you know?

Sample Answers: The elevator is speeding up on (1, 2), (2.5, 3), (3.5, 4) and (4.5, 5). These are the intervals over which the magnitude of velocity is increasing. It is slowing down on (2, 2.5), (3, 3.5) and (4, 4.5). These are the intervals over which the magnitude of velocity is decreasing. Students could also observe the speed with which the red dot on the bar on the left side of the page moves.

Teacher Tip: Reminding students of the directional nature of velocity will help to focus on the absolute value of the change in velocity, rather than the directional value of the change in velocity.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

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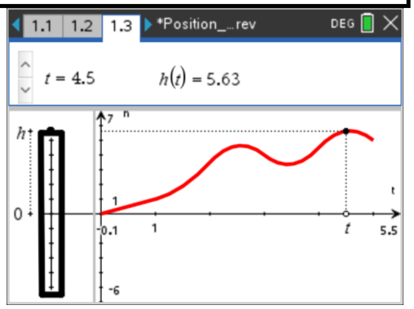
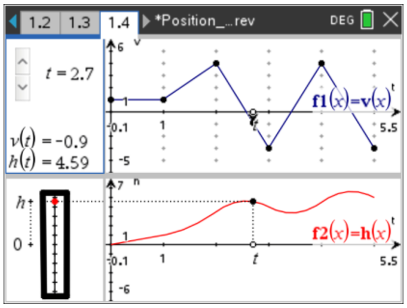


4. This graph shows the height of the elevator vs. time. The height (or position) graph is increasing in some places and decreasing in others. Why?

Sample Answers: When the velocity is positive, this corresponds to the elevator rising (i.e. the height increasing). Negative velocity corresponds to the elevator descending (i.e. height decreasing). Also, the height of the elevator at a given time is the same as the area under the velocity curve over that period of time.

Move to page 1.4.



5. The graph in the top work area shows the height of the elevator over time. The graph in the bottom work area shows the elevator's velocity over time. What are the connections between the height of the elevator and the velocity?

Sample Answers: The height of the elevator is determined by the direction and the magnitude of the velocity up to that time.

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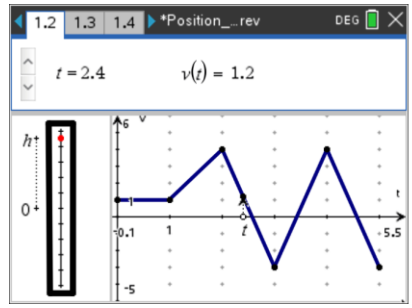
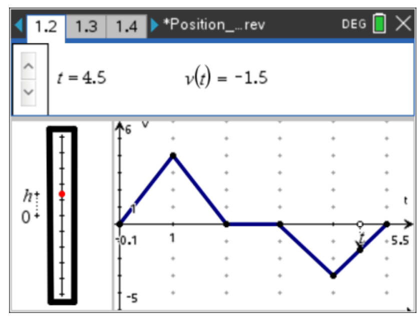


6. Drag the vertices to transform the graph so it models the velocity in the following scenario. Record a sketch of your graph, and explain how it reflects the scenario.

   An elevator begins on the ground floor and starts rising. It speeds up consistently over the first second it rises, to eventually attain a speed of 4 floors /second. It then slows down, slowing consistently over the next second until it stops. It stays stopped for 1 second, and then descends at a faster and faster speed for 1 second. When it is descending at a speed of 4 floors /second, it begins to slow down until it comes to a stop 1 second later.



   __Sample Answers:__ Since the elevator begins from a stop, its velocity at time 0 is 0. It speeds up to reach a speed of 4 floors/second after 1 second, so this gives the point (1, 4). It slows down over the next second, coming to a stop, which means one second later, the velocity is 0, giving the point (2, 0). The elevator stays stopped for 1 second, so for the second between 2 and 3, the velocity graph is constant at 0, with the segment ending at (3, 0). The elevator descends, reaching a speed of 4 floors/ second. Since this is in descent, the velocity is -4. This gives the point (4, -4). Finally, the elevator slows to a stop, ending 5 seconds after it began moving, at a velocity of 0. This gives the point (5, 0) on the velocity graph.

7. What floor is the elevator on when it stops at the end of the scenario? How did you determine this?

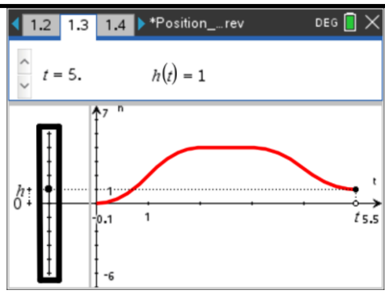
   __Sample Answers:__ The elevator ends on the ground floor. Student reasoning will vary. Students might observe the value of *h* from the bar on the side after 5 seconds. Students also might observe from the velocity graph that the upward and downward velocities are symmetrical, meaning that the total distance risen should equal the total distance the elevator descended.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

8. Is your graph the only graph that can be used to model the scenario in problem 5? Explain your thinking.

   Sample Answers: This is not the only graph that could be used, but it is the only graph that can be constructed given the base graph from which the velocity graph must be built. This is because the base graph requires that the pieces of the graph be linear and that the pieces can change only at integer values of the independent variable. Without these constraints, one could build a velocity graph that met the conditions and was curved, or a velocity graph that met the conditions and was piecewise linear but comprised of more pieces.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.



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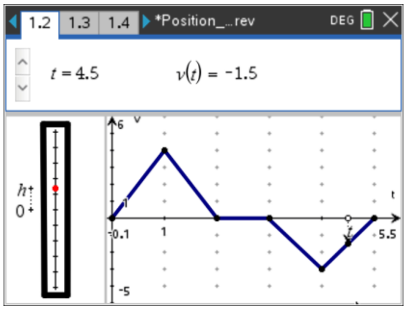
9. According to the position graph, what floor is the elevator on when it stops at the end of the scenario? How does this compare to your response in question 5?

   Sample Answers: It is on the ground floor. This is the same as the response to question 5.



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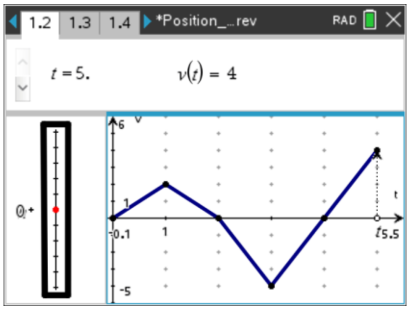
10. Drag the vertices to transform the graph so it models the following scenario. Record a sketch of your graph, and explain how it reflects the scenario.

    An elevator begins on the ground floor. It rises, speeding up and then slowing down, until it comes to the 2nd floor 2 seconds later. It then descends, speeding up and then slowing down again until it reaches 2 floors below ground level 2 seconds later. It then rises, gaining speed as it goes. After 1 second, the elevator is on the ground floor.

Sample Answers: The graph reflects the scenario because it is speeding up, slowing down, rising, and falling according to the constraints of the scenario, and, according to the left bar showing the elevator's height, it puts the elevator on the right floor at the correct time.



TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 5 at the end of this lesson.

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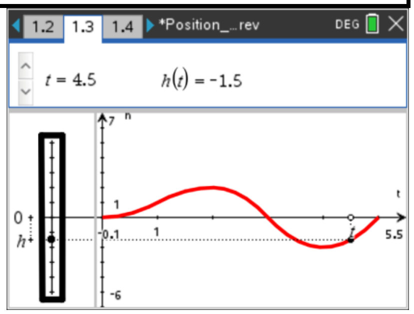

11. Does the height graph on Page 1.3 reflect the scenario described in question 8? If not, what would you need to do to change the velocity graph so the height graph would look correct? Explain.

    Sample Answers: It does reflect the scenario because the height of the elevator was considered while manipulating the velocity graph.

12. In general, how do the velocity and height graphs relate to each other? Explain your thinking.

    Sample Answers: The directional area between the velocity graph and the horizontal axis between 0 and a certain *x*-value is the same as the output of the height graph at that *x*-value. A positive velocity graph corresponds to an increasing height function, and a negative velocity graph corresponds to a decreasing height function. When the magnitude of the velocity graph is increasing, the height function is characterized by steeper slope; and when the magnitude of the velocity graph is decreasing the height function is characterized by a less steep slope.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between velocity and direction of motion of an object.
- The relationship between increase or decrease in a velocity function and the motion of an object.
- How to use a graph to model the velocity of an object in motion.

- How to determine the ending position of an object in motion given information about piecewise linear velocity of the object.

Assessment

- Students write and model their own scenarios regarding velocity of an elevator, and include predictions about corresponding motion of the elevator.
- Students write scenarios about velocity of an elevator and exchange them with other students to graph.
- Students generate and test conjectures about necessary and sufficient information required to graph a velocity function.

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Note 1

Question 2, Name of Feature: Quick Poll

A Quick Poll can be used to determine if students have connected the sign of velocity with the direction of motion.

Note 2

Question 3, Name of Feature: Screen Capture

A Screen Capture can be used to display student solutions to the graph created in Problem 1. This provides opportunities to observe that all graphs meeting the constraints of the problem and the type of graph provided are identical. It also provides an opportunity for a whole group discussion to solidify student understanding of the velocity function. Teachers might want to guide a discussion around the information the velocity graph provides about the motion of the elevator.

Note 3

Question 7, Name of Feature: Quick Poll

A Quick Poll can be used to determine different student solutions for the ending floor. This could be used as a launching point for a discussion of how one determines ending position from velocity, or simply identify for the teacher students to check back in with after Question 7.

Note 4

Question 8, Name of Feature: Screen Capture

A Screen Capture can be used to compare student representations of the velocity graph and provide opportunities for students to evaluate and justify possible graphs.

Note 5

Question 10, Name of Feature: Screen Capture

A Screen Capture can be used to compare student representations and open discussions to evaluate and justify representations.