Transitions

About the Lesson

Students will explore converting rectangular equations to polar form and vice versa. As a result, students will:

- Identify and apply familiar trigonometric identities and circle relationships in making the conversions.

Vocabulary

- rectangular form of an equation
- polar form of an equation
- angle difference identity

Teacher Preparation and Notes

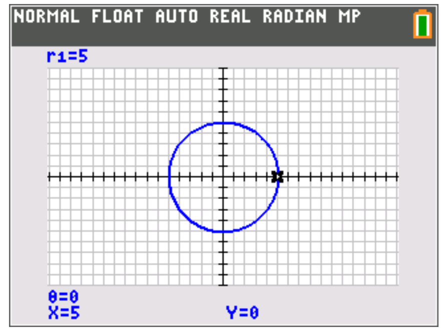
- The first problem explores converting a circle equation to polar form. The second problem engages students in the conversion of a vertical linear equation in polar form to rectangular form.
- In Problem 1 on this activity, students will use the basic trigonometric ratios for sine and cosine as well as the Pythagorean Theorem to convert a rectangular equation to polar form.
- In Problem 2 of this activity, the angle difference identity will be applied to convert a polar equation to rectangular form.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

***** *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

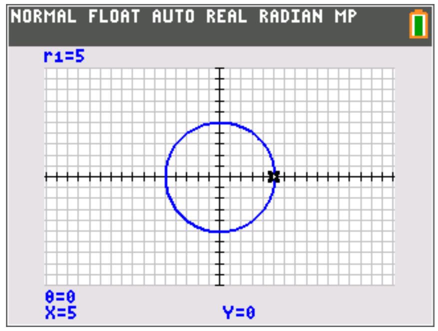
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Transitions_Student.pdf
- Transitions_Student.doc

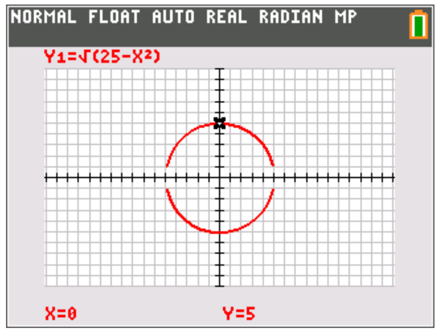
Problem 1 – Converting Rectangular to Polar

Students review some basic relationships relating to the unit circle and apply these relationships in the conversion of a rectangular circle equation to polar form.



$r(\theta) = 5$

Note: Due to the resolution of the graphing window on the graphing calculator, the rectangular form shows gaps at the *x*-axis, but this is because the graph is perpendicular at the axis, and has no slope.



$x^2 + y^2 = 25$

1. What do the polar and rectangular circle equations have in common?

Answer: They are both set equal to some constant. The constant in the polar equation represents the radius of the circle, whereas the constant in the rectangular equation is the square of the radius.

2. Using this diagram, identify three basic equations useful in converting rectangular equations to polar form.

Answers: $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $x^2 + y^2 = r^2$

Students are asked to convert an equation to polar form. Once the conversion is completed, students graph the resulting equation and the initial equation and compare the two equations. The polar graph should appear identical to the rectangular equation graph if the work is done correctly.

It may be necessary to remind students that when the rectangular circle equation is solved for *y*, two equations will result and two must be graphed.

To ensure accurate graphing, make sure the students use **Zoom Standard**, and then **Zoom Square** to ensure equal aspect ratios for both graphs.

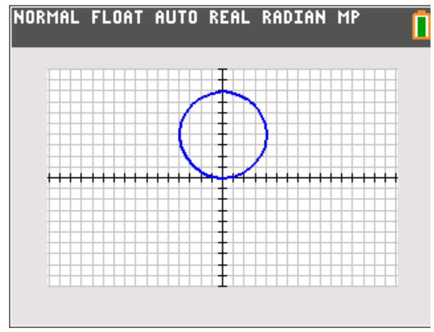
3. Convert $x^2 + (y – 4)^2 = 16$ to polar form using the equations identified above. Show your work in the space provided.

Answer:

$$x^2 + (y - 4)^2 = 16$$
$$r^2\cos^2(\theta) + \left(r \cdot \sin(\theta) - 4\right)^2 = 16$$
$$r^2\left(\cos^2(\theta) + \sin^2(\theta)\right) - 8 \cdot r \cdot \sin(\theta) + 16 = 16$$
$$r^2(1) = 8 \cdot r \cdot \sin(\theta)$$
$$r = 8\sin(\theta)$$

4. Test your results by graphing both the rectangular and polar forms. If the results are equivalent, as long as the graphing window is set at the same values, the graphs will appear the same. Make sketches of the initial rectangular equation and the polar equation obtained in the previous exercise.

Answer: Both graphs should look the same. See screenshot to the right.



Problem 2 – Converting Polar to Rectangular

Next, students convert a polar equation to rectangular form using the angle difference identity for cosine. As in Part 1, substitution involving familiar identities is useful in making the conversion. Students will also graph their results to check the accuracy of their solutions.

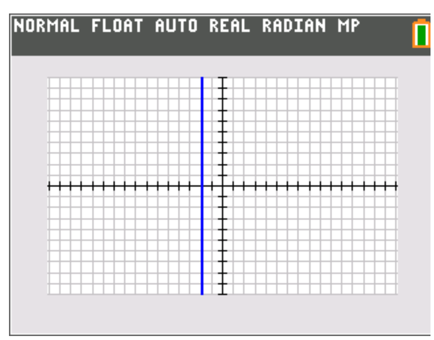
5. Convert the polar equation $2 = r \cdot \cos\left(\theta + \pi\right)$ to rectangular form. Show your work in the space provided.

Answer:

$$2 = r \cdot \cos\left(\theta + \pi\right)$$
$$2 = r\left(\cos(\theta)\cos(\pi) - \sin(\theta)\sin(\pi)\right)$$
$$2 = r\left(-1 \cdot \cos(\theta) - 0 \cdot \sin(\theta)\right)$$
$$2 = -r \cdot \cos(\theta)$$
$$-2 = x$$

6. Make sketches of the initial rectangular equation and the polar equation obtained in the previous exercise on the pair of axes below.

Answer: Both graphs should look the same. See screenshot to the right.



Additional Practice

7. Write the polar form of each of the following equations:

a. $x^2 + y^2 = 64$

Answer: $r = 8$

b. $(x – 2)^2 + y^2 = 4$

Answer: $r = 4\cos\theta$

c. $x = –5$

Answer: $r \cdot \cos\theta = –5$ or $r = –5\sec\theta$

d. $x^2 – y^2 = 1$

Answer: $r^2 = \dfrac{1}{2\cos^2\theta - 1}$ or

$r^2 = \dfrac{1}{1 - 2\sin^2\theta}$ or $r^2 = \sec(2\theta)$

8. Write the rectangular form of each of the following polar equations:

a. $r = 3$

Answer: $x^2 + y^2 = 9$

b. $r = 3\sin\theta$ (Hint: Multiply each side by r first.)

Answer: $x^2 + y^2 = 3y$

c. $6 = r \cdot \cos\left(\theta - \dfrac{\pi}{4}\right)$

Answer: $6\sqrt{2} = x + y$

d. $r = 3\sec(\theta + 60°)$

Answer: $x - \sqrt{3} \cdot y = 6$