# Vector Dot Products

TI-Nspire™          Investigation          Student          30 min

## Introduction

A formula exists to compute the dot product of two vectors, but what does it mean?  Can we 'see' the dot product? In this activity you will be able to manipulate two vectors $a$ and $b$, and see the impact on the dot product. When is the dot product zero? When is the dot product negative?

## Instructions

Open the TI-Nspire file "Dot Product" and navigate to page 1.2

Use the mouse to move over the head of vector $b$ until the word "Point" appears. When the word "Point" appears (shown opposite) either:

- Click and hold the centre of the navigation pad for 1 second
  
  OR
- Press Ctrl + Click

Vector $b$ can now be moved. Press [Esc] to release the vector. Note that the magnitude (length) of vectors $a$ and $b$ are shown at the base of the page.
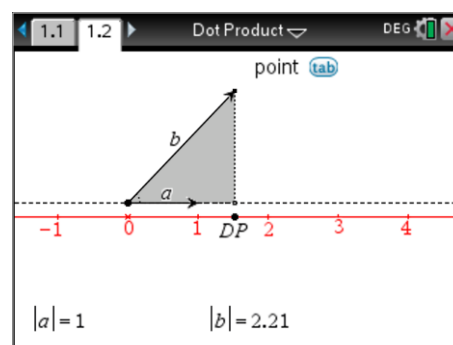


## Question 1

Change the magnitude of vector $a$ and $b$ and also the angle between the two vectors.

a)  Does vector $a$ affect the dot product?

Yes – the magnitude of vector $a$ is directly proportional to the dot product.

b)  Does vector $b$ affect the dot product?

Yes – the magnitude of vector $b$ is directly proportional to the dot product.

c)  Does the angle between vectors $a$ and $b$ affect the dot product?

Yes – If the vectors are parallel (and in the same direction) then the dot product is equal to $|a||b|$. If the vectors are perpendicular then the dot product is zero. If the angle between the vectors is between 90° and 270° then the dot product is negative.

### Teacher Notes:

The purpose of the exploration is for students to identify the important characteristics so that later when they learn the formula for the dot product of two vectors they have a natural expectation that the magnitude of each vector and the angle between the two vectors will be included in the formula. Students should at least identify circumstances when the dot product is zero and also when it is positive / negative.

*For each of the above, describe the affect in detail. For example can the dot product be negative or zero?*

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## Question 2

Adjust vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ so the dot product is approximately equal to 2. Determine at least two different values for the magnitude of the two vectors and the approximate angle between them.

Answers will vary.

**Teacher Notes:**

The purpose of this question is for students to focus specifically on the relative magnitudes of the two vectors and also on the angle. If students have not made any connections, generating multiple results for a dot product equal to 2 will take a considerable amount of time.

## Question 3

In the diagram shown opposite vector $\underset{\sim}{a}$ (OA) has a magnitude of 1.5, this can be written as: $\left|\underset{\sim}{a}\right| = 1.5$.

Vector $\underset{\sim}{b}$ (OB) has a magnitude of 3, this can be written as: $\left|\underset{\sim}{b}\right| = 3$.

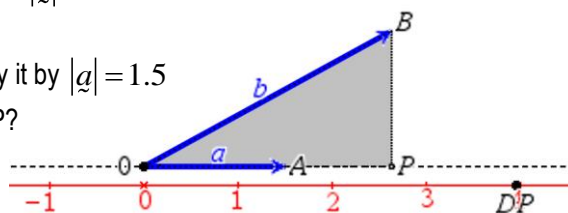The angle between the two vectors is approximately 28°.
Use trigonometry to calculate the length of OP and multiply it by $\left|\underset{\sim}{a}\right| = 1.5$
How does this compare to the value for the dot product DP?



OP = $3 \times \cos(28) \approx 2.65$

OP $\times 1.5 \approx 4$

The result compares favourably to the visual representation of the dot product (DP) $\approx 4$.

**Teacher Notes:**

The purpose of this question is for students to see the simple connection between basic trigonometry and vectors, particularly vectors in two dimensions. It is not trivial that the same result also holds true for vectors in three dimensions.

## Question 4

Repeat Question 3 with some other values of your own.
The angle between the two vectors can be measured using: **Menu > Measure > Angle**.

Answers will vary.

**Teacher Notes:**

The purpose of this question is for students to begin to formulate (through repetition) that the dot product of the two vectors is equal to: $\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos(\theta)$ . For students that do not arrive at this formula it will need to be provided to students. Students should then reflect on the activity in order to make the connections.

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